Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 52(12) (2021) 845-848
DOI: 10.22060/mej.2019.16834.6451

Design of Three-dimensional Robust Guidance Law Using Adaptive Dynamic Programming with Input Saturation Constraint

S. Khan Kalantary, I. Izadi*, F. Shekholeslam

Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan, Iran

ABSTRACT: In this paper, a three-dimensional robust guidance law for an interceptor considering input saturation and first-order dynamic for the autopilot system is designed. To attain this goal, first, modeling of the system in three-dimensional spherical coordination using engagement basics has been derived and after that, the appropriate cost function for a collision of interceptor and target considering actuator constraints and in absence of target movement information has been formulated. According to robust control literature for achieving this type of guidance laws, Hamilton-Jacobi-Isaacs differential equation inequality should be solved which unfortunately does not have a closed-form solution in our problem. Therefore, to overcome this challenge, using adaptive dynamic programming theory for solving acquired Hamilton-Jacobi-Isaacs, an algorithm for designing robust guidance law has been presented. Simplification of the differential inequality and also satisfying the robustness of the controller to different unknown target movemnts, are the most important features of the proposed algorithm. Various simulations for targets with different movements and comparison of the proposed method with conventional augmented proportional navigation, show the effectiveness of the designed three-dimensional robust guidance law.

Review History:

Received: 27 Jul. 2019
Revised: 7 Sep. 2019
Accepted: 22 Sep. 2019
Available Online: 14 Oct. 2019

Keywords:

Guidance law
Interception
Robust control
Adaptive dynamic programming

1- Introduction

For interceptors, The main goal of designing guidance laws in terminal phase is to minimize the distance between the interceptor and the target. High maneuverable targets, dynamical complexities and performance limitations, and physical constraints, are severe challenges in designing a proper guidance law.

Proportional Navigation (PN) performance, as one of the most successful guidance laws for interceptors, diminishes against maneuvering targets significantly. H_∞ control, in which knowing target acceleration is not necessary, has become more popular in recent years [1-4]. Considering target acceleration as an external disturbance, the problem can be formulated as a zero-sum game, which needs a Hamilton - Jacobi - Isaacs (HJI) differential equation to be solved. Because of complexity due to nonlinear dynamics and constraints, a closed-form solution does not exist.

In this research, the goal is to design a robust controller for an interceptor, considering input saturation and first-order dynamics for the autopilot system. Unlike most of the papers which used two-dimentional model of engagement, here the persuit-evasion is modeled in the three-dimentional spherical coordinates. Considering target acceleration as an external disturbance and including input saturation constraints, a proper cost function is derived . For solving the resulted zero-sum game through its HJI differential equation, an Adaptive Dynamic Programming (ADP) method is used. The proposed algorithm will approximate the cost function of HJI inequality using neural network general function approximation ability. The assumed neural network weightigs are calculated by an offline method. Finally, a comparison is made between the proposed method and Augmented Proportional Navigation (APN) guidance law using different target maneuvers.

*Corresponding author's email: iman.izadi@cc.iut.ac.ir

2- Methodology

Pursuit-evasion geometry of target and interceptor in the spherical coordinate system is illustrated in Fig. 1. In this figure, r is the relative position along the line of sight [5]. By differentiating from r, one has:

$$\dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\theta}\cos(\varphi)\vec{e}_\theta + r\dot{\varphi}\vec{e}_\varphi \tag{1}$$

in which θ, φ, and r are elevation, azimuth, and relative position, respectively. After differentiation from Eq. (1), we have:

$$\begin{cases} \ddot{r} - r\dot{\varphi}^2 - r\dot{\theta}^2\cos^2(\varphi) = w_r - u_{Mr} \\ r\ddot{\theta}\cos\varphi + 2\dot{r}\dot{\theta}\cos(\varphi) - 2r\dot{\varphi}\dot{\theta}\sin(\varphi) = w_\theta - u_{M\theta} \\ r\ddot{\varphi} + 2\dot{r}\dot{\varphi} + r\dot{\theta}^2\cos(\varphi)\sin(\varphi) = w_\varphi - u_{M\varphi} \end{cases} \tag{2}$$

where w_r, w_θ, and w_φ are acceleration components of the target, and u_{Mr}, $u_{M\theta}$, and $u_{M\varphi}$ are acceleration components of the interceptor. To compensate for the difference between required and applied acceleration, first-order dynamics are considered for autopilot.

Copyrights for this article are retained by the author(s) with publishing rights granted to Amirkabir University Press. The content of this article is subject to the terms and conditions of the Creative Commons Attribution 4.0 International (CC-BY-NC 4.0) License. For more information, please visit https://www.creativecommons.org/licenses/by-nc/4.0/legalcode.

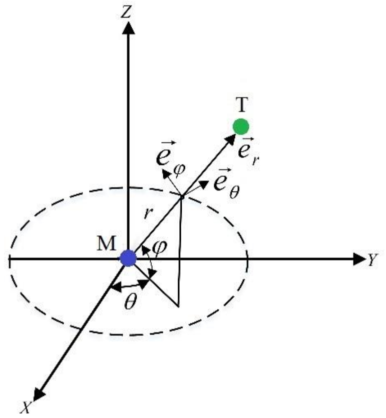

Fig. 1. Pursuit-evasion geometry in the three-dimensional spherical coordinate system

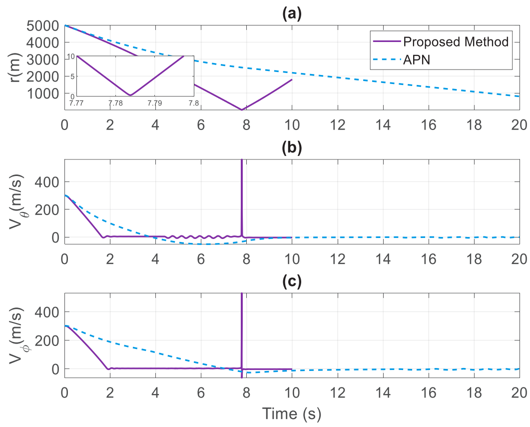

Fig. 3. Simulation results for robust guidance law and APN for a target with ramp maneuver: (a) distance between target and interceptor, (b) elevation angle rate, (c) azimuth angle rate

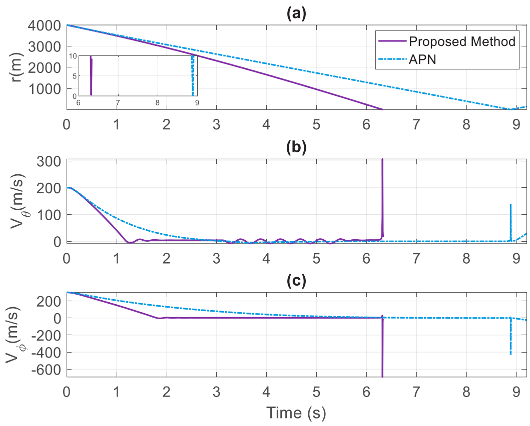

Fig. 2. Simulation results for robust guidance law and APN for a target with step maneuver: (a) distance between target and interceptor, (b) elevation angle rate, (c) azimuth angle rate.

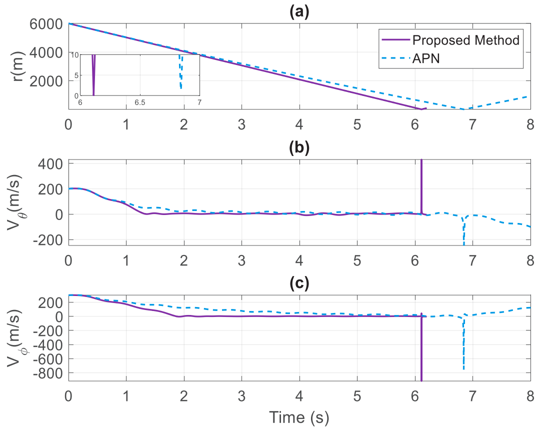

Fig. 4. Simulation results for robust guidance law and APN for a target with sine maneuver: (a) distance between target and interceptor, (b) elevation angle rate, (c) azimuth angle rate

Formulation of a zero-sum game is depicted as follows:

$$\min_u \max_d V(x,u,d) : \int_0^\infty J(x,u,d)\,d\tau$$
$$= \int_0^\infty \left(h(x)^T h(x) + u^T u - \gamma^2 d^T d\right) dt$$
$$s.t.\ \begin{cases} \dot{x}(t) = f(x) + g(x)u(t) + k(x)d(t) \\ |u_r(t)| \le \beta_1 \\ |u_\theta(t)| \le \beta_2 \\ |u_\phi(t)| \le \beta_3 \end{cases} \tag{3}$$

in which, the disturbance is trying to increase the cost function, while the control aims at decreasing it. Using the following quasi-norm of reference [6]:

$$\|u\|_q^2 = 2\int_0^u \phi^{-1}(v)\mathrm{d}v = \sum_{k=1}^m 2\int_0^{u_k} \phi^{-1}(v)\mathrm{d}v \tag{4}$$

The problem is transformed to an unconstrained optimization which should be solved through the following differential equation:

$$V_x^T f - V_x^T g\phi\left(\frac{1}{2}g^T V_x\right) + h^T h$$
$$+2\int_0^{-\phi\left(\frac{1}{2}g^T V_x\right)} \phi^{-1}(v)\mathrm{d}v + \frac{1}{4\gamma^2} V_x^T k k^T V_x = 0 \tag{5}$$

Here, $V = \int_t^\infty J(x,u,d)\,\mathrm{d}\tau$ and $V_x = \partial V / \partial x$ are the value function and its partial derivative respectively. To solve Eq. (5), the proposed iterative algorithm of reference [7] is applied. Having an initial stabilizing controller of u_j, one solves for:

$$\left(V_{xj}^i\right)^T \left(f(x) + g(x)u_j + kd^i\right) + h^T h$$
$$+2\int_0^{u_j} \phi^{-T}(v)\mathrm{d}v - \gamma^2 \left\|d^i\right\|^2 = 0 \tag{6}$$

where, the worst case external disturbance is calculated as $d^{i+1} = \left(1/2\gamma^2\right) k^T V_{xj}^i$. After convergence of V_j^i to V_j^* in the inner loop, the external controller is updated to $u_{j+1} = -\phi\left(\frac{1}{2} g^T V_{xj}^*\right)$ in the outer loop. For computational reasons, the value function is approximated by the following neural network [21]:

$$\hat{V}_j^i(x) = \sum_{k=1}^{L} w_{j,k}^i \sigma_k(x) \tag{7}$$

where, $w_{j,k}^i$ and $\sigma_k(x)$, are weightings and activation functions respectively.

3- Results and Discussion

The comparison between initial conditions for the proposed guidance law and APN are considered for three different target accelerations: step, ramp, and sine.The obtained results are shown hereunder:

4- Conclusion

In this article, a robust guidance law of an interceptor with first-order autopilot dynamics and saturation constraints was developed. The guidance law design was formulated as a constrained zero-sum and, an ADP based method is proposed to solve the complicated nonlinear HJI equation. The most important advantage of the proposed method over the APN method is that the former does not need the target acceleration to be known. Simulation results for various target acceleration profiles, confirm the robustness of the proposed algorithm in comparision to APN.

References

[1] C.-D. Yang, H.-Y. Chen, Nonlinear H robust guidance law for homing missiles, Journal of Guidance, Control, and Dynamics, 21(6) (1998) 882-890.

[2] A.V. Savkin, P.N. Pathirana, F.A. Faruqi, Problem of precision missile guidance: LQR and H∞ control frameworks, IEEE Transactions on Aerospace and Electronic Systems, 39(3) (2003) 901-910.

[3] C.-S. Shieh, Tunable H∞ robust guidance law for homing missiles, IEE Proceedings-Control Theory and Applications, 151(1) (2004) 103-107.

[4] C.D. Yang, H.Y. Chen, Three-dimensional nonlinear H∞ guidance law, International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal, 11(2) (2001) 109-129.

[5] F.P. Adler, Missile guidance by three-dimensional proportional navigation, Journal of Applied Physics, 27(5) (1956) 500-507.

[6] M. Abu-Khalaf, F.L. Lewis, Nearly optimal control laws for nonlinear systems with saturating actuators using a neural network HJB approach, Automatica, 41(5) (2005) 779-791.

[7] M. Abu-Khalaf, F.L. Lewis, J. Huang, Policy iterations on the Hamilton–Jacobi–Isaacs equation for H∞ state feedback control with input saturation, IEEE Transactions on Automatic Control, 51(12) (2006) 1989-1995.