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Free and Forced Vibration Analysis of Piezoelectric Patches Based on Semi-Analytic Method of Scaled Boundary Finite Element Method

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ABSTRACT: Development of a precise mathematical model of piezoelectric patches plays an important role in comprehending their operational mechanisms as well as developing new techniques based on their coupled electro-mechanical behavior. While, high computational cost of available numerical methods which are able to simulate vibrational behavior of piezoelectric patches, especially at high frequencies, is considered as a serious challenge in this area. The purpose of this study is to use a novel semi-analytical method, called Scaled Boundary Finite Element Method, to analyze free and forced vibration of piezoelectric patches. In order to evaluate the accuracy of this method in modeling of different problems occurred in structural health monitoring and fracture mechanics, the free and forced vibration of a piezoelectric patch, a piezoelectric patch attached to an aluminum structure, a piezoelectric patch with a circular hole and a cracked piezoelectric patch was analyzed as four case studies. Comparison of convergence rate of scaled boundary finite element method and finite element method indicates that the former provides exact results with much less degrees of freedom. In addition, proper matching of results demonstrates the capability of scaled boundary finite element method to model a variety of problems accurately at a very low computational cost.

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1- Introduction

Recently, employing miniaturized piezoelectric patches in various branches of science and engineering such as structural health monitoring, has become widespread due to their unique characteristics including: simultaneous sensing/actuating capability, light weight high strength, non-intrusivity and low cost. Developing an accurate mathematical model to describe the behavior of piezoelectric patches, plays an important role in comprehending their coupled electromechanical behavior. So far, several numerical methods have been used to analyze vibration of piezoelectric patches such as: Finite Difference Method (FDM), Boundary Element Method (BEM), Finite Element Method (FEM), and Spectral Finite Element Method (SFEM) [1-3]. Despite some specific advantages of these methods, they confronts practical constraints dealing with multi-material systems, discontinuous configurations and complex process of mesh size tuning for simulation of high frequency vibrations which leads to overwhelming computational cost [1, 3-4].

Scaled Boundary Finite Element Method (SBFEM) is a relatively novel numerical method to solve governing partial differential equations of a variety of engineering systems [5]. In this method, the intended domain is first split into non-overlapping subdomains (sometimes called S-elements) to be able to sight each point on their boundaries directly known as scaling requirements. Discretizing only the boundary of each S-element as in BEM and treating the problem in radial direction rigorously, classifies SBFEM as a semi-analytical approach. This is while, there is no fundamental solution is necessary unlike BEM [6]. So far, SBFEM has been implemented in various fields including: elastodynamics and fracture mechanics.

Despite the considerable efforts which has been made toward implementation of SBFEM in various engineering areas, it has not been used to analyze elastodynamic behavior of piezoelectric patches at high frequencies. The purpose of this study is to investigate the capability of 2 dimensional coupled field SBFEM to model high-frequency vibration of piezoelectric patches in both time and frequency domain. The SBFEM results were also compared with their corresponding FEM results in terms of convergence rate, accuracy and computational cost.

2- Methodology

According to virtual work principle [7], the governing equation of SBFEM is obtained as follows [8]:

$$\begin{aligned}&\left[E^{0}\right]\xi^{2}\bar{u}(\xi)_{,\xi\xi}+\left(\left[E^{0}\right]-\left[E^{1}\right]+\left[E^{1}\right]^{T}\right)\xi\bar{u}(\xi)_{,\xi}\\&-\left[E^{2}\right]\bar{u}(\xi)-M^{0}\xi^{2}\ddot{\bar{u}}(\xi)=0\end{aligned}\tag{1}$$

Which can be reformulated in terms of dynamic stiffness in frequency domain as follows [8]:

$$\begin{aligned}&\left(\left[S(\omega)\right]-\left[E^{1}\right]\right)\left[E^{0}\right]^{-1}\left(\left[S(\omega)\right]-\left[E^{1}\right]^{T}\right)\\&+\omega\left[S(\omega)\right]_{,\omega}-\left[E^{2}\right]+\omega^{2}\left[M_{0}\right]=0\end{aligned}\tag{2}$$

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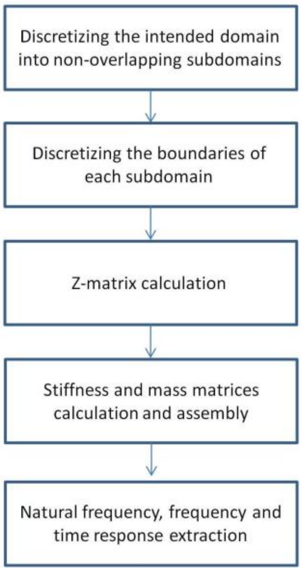

Fig. 1. Solution procedure of SBFEM

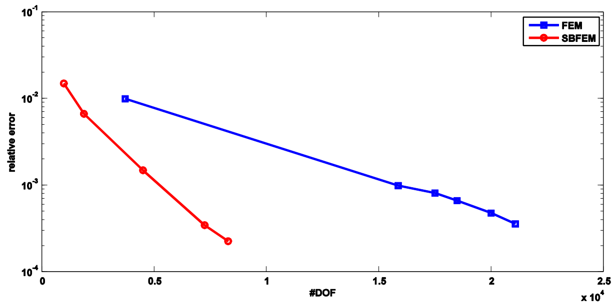

Fig. 2. Averaged relative error of the first 50 eigen-frequencies the perforated piezoelectric patch

To solve the above equation, the dynamic stiffness matrix is replaced by a continued fraction expansion in terms of frequency. The expansion order is closely related to the intended frequency range. Increasing the order of continued fraction expansion, improves the accuracy, especially at high frequencies. This is while, this approach is accompanied by introduction of additional auxiliary variables which will increase the number of Degrees Of Freedom (DOF) and thus, computational cost. To overcome this drawback, employment of finer subdomains in order to discretize the intended domain and ignoring the high-order terms in dynamic stiffness matrix expansion is recommended [9]:

$$\left[S\left(\omega\right)\right]=\left[K\right]-\omega^{2}\left[M\right] \tag{3}$$

where [k] and [M] are stiffness and mass matrices of S-element, and are obtained through solving an algebraic Riccati equation and a Lyapunov equation, respectively. Collectively, the solving procedure of SBFEM for elastodynamic analysis of piezoelectric patch is summarized here:

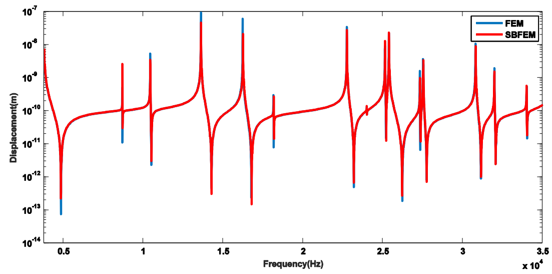

Fig. 3. Frequency response of the perforated piezoelectric patch (frequency range of 0-35 kHz)

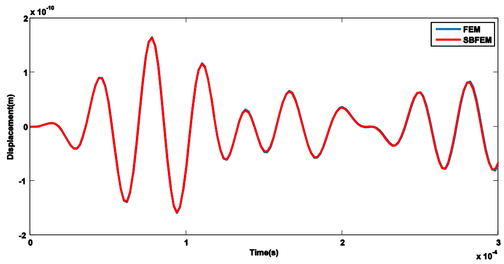

Fig. 4. Transient response of the perforated piezoelectric patch (0 - 300)

3- Results and Discussion

Fig. 2 compares the convergence rate of SBFEM and FEM in terms of first 50 eigen-value deviation index. According to steeper slope of SBFEM graph, its convergence rate is much greater than FEM, which indicates the lower number of DOF required to discretize the intended domain with SBFEM. Figs. 3 and 4 depict the system response to high frequency excitation in the form of point displacement in y direction in frequency and time domains, respectively. These results correspond to the system response to chirp (frequency analysis) and 5 cycle Hanning windowed tone burst (temporal analysis) excitation. The great agreement of the results demonstrates the potential of SBFEM to simulate high frequency standing and propagating waves behavior in perforated (source of discontinuity and stress concentration) piezoelectric patch; however, with much simpler mesh generation process and much less computational cost (the SBFEM DOF is about one-third of FEM).

4- Conclusions

The elastodynamic behavior of piezoelectric patches at high frequencies were investigated using SBFEM. In order to evaluate the accuracy of SBFEM, the free and forced vibration of a piezoelectric patch, a piezoelectric patch attached to an aluminum structure, a piezoelectric patch with a circular hole and a cracked piezoelectric patch was analyzed as four case studies. Comparison of convergence rate of SBFEM and FEM indicates that the former provides exact results with much less DOF. In addition, proper matching of results demonstrates the capability of SBFEM to model a variety of problems accurately at a very low computational cost.

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