On the deviation from cyclic symmetry in a spinning ring under moving electromagnetic loading

Saber Sheikh Khoshkar, Seyedali Sina*, Alireza Shaterzadeh

Department of Mechanical and Mechatronics Engineering, Shahrood University of Technology

ABSTRACT
Vibration problems are of great importance in design and construction of electric machines which affect both of mechanical and electrical properties of these machines. In this paper, the effects of both perfect and perturbed conditions of cyclic symmetry on the vibration behavior of spinning ring under moving electromagnetic loading have been investigated. Euler-Bernoulli beam assumptions have been implemented in the modeling of structure and electromagnetic loading has been modeled with discrete springs. Using Hamilton’s principle, the governing equations of in-plane vibrations of spinning ring have been extracted. Eigen analysis of the system has been extracted using perturbation methods. The obtained results show the condition of instability for a precise value of ring angular speed versus support speed in different vibration modes. The effects of time variation of spring’s stiffness and the variation of connection angle between springs and ring on the in-plane vibration of spinning ring, have been investigated extensively. It is shown that the deviation from cyclic symmetry could eliminate the mode splitting phenomenon in some cases. The obtained results are expected to offer better predictions of the vibrational behavior of spinning rings structures under moving loads in general, and in the design of electric machines, in particular.

KEYWORDS
In-plane vibration, Spinning ring, cyclic symmetry, Moving electromagnetic loading, Mode splitting

* a.sina@shahroodut.ac.ir
1. Introduction

Symmetric rings exposed to rotating loads have wide applications in engineering. In electric machines, rings are subjected to rotating electromagnetic in-plane forces. In order to achieve an efficient design, the vibrational behavior of these machines should be studied. In this context, Canchi and Parker [1] studied the rotating ring with moving and time-varying springs. Sun et al [2] investigated the phenomenon of splitting in the vibration mode shape of a symmetrical rotating ring. Zhao and Wang [3] investigated the parametric instability induced by the magnetic load in the permanent magnet motors.

2. Methodology

The spinning ring under moving electromagnetic loading is modeled according to Figure 1.

![Figure 1. Schematic of spinning, cyclic symmetric ring under moving electromagnetic loading](image)

In the next sections, the governing equations of motion of the cyclic symmetric elastic ring are presented. The equations of motion are derived using Hamilton principle and the Euler-Bernoulli beam theory have been utilized. The non-dimensional form of the equation of motion has been stated in Eq.1.

\[
M \frac{d^2 u}{dt^2} + G \frac{\partial u}{\partial t} + \left[ D + K^{(0)} + \varepsilon K^{(1)} \right] u = 0, \tag{1}
\]

Then, the frequency analysis of the cyclic symmetric ring and spring system is performed using perturbation theory. Both the stationary and non-stationary cases corresponding to the absence or presence of time-varying coefficients in the equation of motion, respectively, have been considered. The electromagnetic load has been modeled by spring. Hence, variation of electromagnetic force parameters (see Fig.1) could affect the cyclic symmetry in the system. The results obtained in the present study show the importance of this issue in the vibration behavior of electric machines. The effects of deviation from the cyclic symmetry in the ring are investigated using perturbation method. The splitting phenomenon has been observed in some cases which been studied in detail. The obtained results indicate high influence of the design parameters on the vibration behavior and stability of the system.

3. Results and Discussion

Analytical results show that vibration mode splitting occurs for the cases with the integer values of \(2n/N\), which \(n\) and \(N\) correspond to the vibration mode number and the order of cyclic symmetry in the system, respectively. The results are obtained for both cases of cyclic symmetric and non-cyclic symmetric ring and spring systems in relative speeds of \(\Omega_m = \Gamma/n\) and \(\Omega_m \neq \Gamma/n\). According to the obtained results, vibration mode splitting occurs in the relative speed \(\Omega_m = \Gamma/n\) in the cyclic symmetric system. For the sake of brevity, the results related to the relative speed of \(\Omega_m \neq \Gamma/n\) are not presented here.

3.1 Cyclic symmetric ring and spring system with time variable stiffness

Figures 2-a and 2-b show the time evolution of the imaginary and real parts of the system eigenvalues. The results are corresponding to the integer value of \(2n/N\) (\(n\) and \(N\) are circumferential wavenumber and number of rotating supports, respectively) and \(\Omega_m = \Gamma/n\) where \(\Gamma\) is a function of ring of speed and \(n\). The Figure 2-a depicts a periodic amplitude of the splitting while Figure 2-b shows that the splitting causes instability in the system.
Figure 2. Time variations of the imaginary and real parts of cyclic symmetric ring and variable stiffness spring system, Sine and Cosine vibration modes

3.2 Non-cyclic symmetric ring and variable stiffness spring system

In this section, the time variation of spring stiffness leads to the deviation from cyclic symmetry in the ring-spring system. In Fig. 3, the time history of the imaginary and real parts (Figures 3-a and 3-b, respectively) of the system eigenvalues have been shown for the case of integer value of $2n/N$, and $\Omega_{rel} = \Omega/n$. Due to the deviation from cyclic symmetry, mode splitting will be disappeared and the frequencies of the sine and cosine modes converge to fixed values. Also, the real part of the system eigenvalue becomes zero, implying that the system is stable as shown in Figure 3-b.

Figure 3. Time variations of the imaginary and real parts of non-cyclic symmetric ring and variable stiffness spring system, Sine and Cosine vibration modes

4. Conclusion

In this paper, the effects of both perfect and perturbed conditions of cyclic symmetry on the vibration behavior of spinning ring under moving electromagnetic loading have been investigated. The effect of deviation from the cyclic symmetry on the instability of the system has been investigated. Changes in spring stiffness in the form of harmonics and sawtooth without cyclic symmetry could eliminate the mode splitting phenomenon and instability of the system. Similar effect has been reported in the case of variation of inclined angle between ring and springs.

5. References