Modeling of vibro-acoustic modulation induced by non-linear contact in the Euler-Bernoulli beam using the Fourier spectral element

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ABSTRACT

Exploiting the nonlinear nature of structural damage through piezoelectric patches is one of the latest concepts of early detection of damage. Support loosening as one of the prevalent defects in engineering systems, is a source of contact acoustic nonlinearity. Vibro-acoustic modulation has proven to be a promising method for revealing structural nonlinearity characteristics. Requiring a large number of cycles to be solved in the time domain to reach the steady-state before evaluating the results in the frequency domain, makes using existing computational methods such as FEM to model this phenomenon very expensive. This paper is concerned with a novel numerical method called Fourier spectral element to investigate the vibro-acoustic modulation in the Euler-Bernoulli beam caused by contact nonlinearity. Three piezoelectrics were attached to the beam as the probe, pump and sensor. The numerical outcomes are compared against the experimental results to check the validity of the developed method. The results show that the Fourier spectral element not only provides a high convergence rate but is also capable of simulating the phenomenon of modulation with sufficient accuracy.

KEYWORDS

Structural health monitoring, vibro-acoustics modulation, contact acoustics nonlinearity, piezoelectric patches, Fourier spectral element

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1. **Introduction**

The use of piezoelectric patches is widespread in monitoring the health of various industrial structures and optimizing their maintenance process. Online monitoring methods use linear or nonlinear principles to identify defects [1-4]. Early detection of structural defects is one of the advantages of nonlinear methods over linear ones. Vibro-acoustic modulation method is one of the non-linear health monitoring methods of structures [1].

In a general classification, the types of nonlinear behavior of structures are divided into classical and non-classical categories. Nonlinear contact includes any interaction of solid surfaces with each other due to cracking, separation and loosening of the screw or support and can be classified as non-classical nonlinearity. So far, various experimental methods have been proposed to identify defects with non-classical nonlinear effects, many of which fall into the "pump-carrier" category.

In [5], a finite element model was used to determine the appropriate frequency for pump excitation. The open and closed state of the crack due to the excitation of the pump with adjusted frequency caused the pump wave to combine with the carrier wave. In [6], the acoustic and vibrations modulation caused by contact at the boundary have been investigated using the finite element method. Simulation of the modulation phenomenon in a composite laminate by Singh et al. is one of the few other efforts in this field [7].

The purpose of this paper is to investigate the nonlinear vibrations of the host structure due to loosening and contact at its support for early and optimally monitor of its health. For this purpose, experimental tests were designed on beams equipped with piezoelectric patches in healthy and defected states. The effect of pump frequency on increasing the ability to detect damage is one of the points of interest in this article. In order to understand the physical mechanism governing the modulation phenomenon, numerical modeling is presented using the Fourier spectral element method. The electromechanical behavior of piezoelectric patches is considered in the developed model. Then, contact mechanics was introduced to the developed formulation as a nonlinear factor governing the problem. The idea of an elastic and loose substrate between the host structure and the rigid boundary was used to model the nonlinear factor. Due to the transient nature of the problem under study, the governing equations are analyzed in the time domain and with the help of the explicit central difference algorithm. The process of signal processing and extraction of damage-sensitive indices in the frequency domain are among the other perspectives of the present paper.

2. **Methodology**

The schematic of the under study Euler-Bernoulli beam is shown in Figure 1. To investigate the contact at the boundary, the effect of the loosing support is modeled as an elastic substrate using transverse springs, and both transverse and longitudinal vibrations are considered in the modeling.

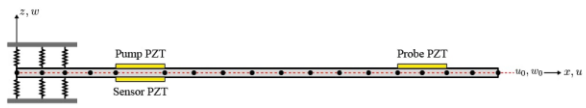

Figure 1 Beam geometry with elastic foundation

The Fourier sine series of a continuous arbitrary function f (x) in the interval $[0,L]$ is:

$$f(x)=\sum_{m=1}^{\infty}A_m sin\left(\frac{m\pi}{L}x\right) \qquad 0<x<L \tag{1}$$

To calculate the stiffness and mass matrices, first the potential and kinetic energy of the beam with the elastic substrate are calculated. The kinetic energy and potential of the i^{th} beam element with elastic substrate are as follows:

$$\begin{aligned} T &= \frac{1}{2}\int_{L_{i-1}}^{L_i}\left[\rho A\left(\frac{\partial \bar{u}_i}{\partial t}\right)^2+\rho A\left(\frac{\partial \bar{w}_i}{\partial t}\right)^2\right]dx \\ U &= \frac{1}{2}\int_{L_{i-1}}^{L_i}\left[EA\left(\frac{\partial \bar{u}_i}{\partial x}\right)^2+EI\left(\frac{\partial^2 \bar{w}_i}{\partial x^2}\right)^2+k\bar{w}_i^2\right]dx \end{aligned} \tag{2}$$

Due to the non-linear nature of the contact, to investigate this effect, the beam area with the elastic substrate should be studied. The terms of contact known as Hertz-Signorini-Moreau are:

$$\begin{cases} g_N^i \geq 0 \\ \lambda_N^i \geq 0 \qquad i=1,2,..,s \\ g_N^i\lambda_N^i=0 \end{cases} \tag{3}$$

3. **Results and Discussion**

The vector of external electric load was calculated and applied to the degrees of freedom corresponding to the pump and carrier actuators. The carrier frequency was 100 kHz and the pump frequency was 22 and 27 kHz.

Figure 2 shows the simulation results for a carrier frequency of 100 kHz and a 22 kHz pump in a healthy structure. According to the figure, due to the linear nature of the structure in the desired frequency range, only the frequency of 100 kHz has occurred in the response spectrum. Figure 3 shows the simulation results for the carrier frequency of 100 kHz and the 22 kHz pump in the defected structure.

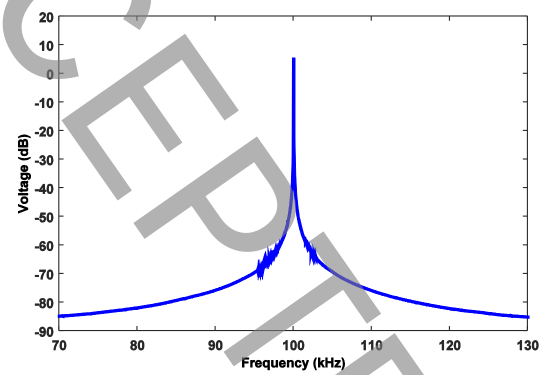

Figure 2 Spectrum of structural response to pump (22 kHz) and probe (100 kHz) excitations in the healthy state

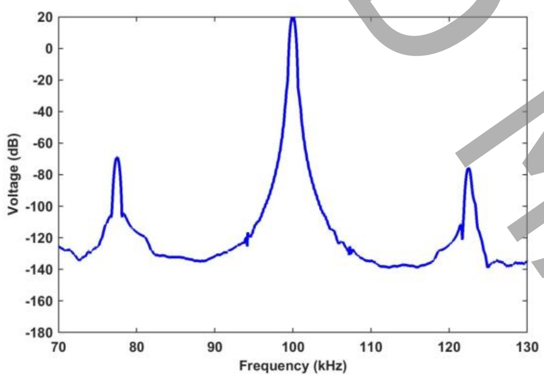

Figure 3 Spectrum of the structural response to pump (22 kHz) and probe (100 kHz) excitations in the damage state

According to the results, in addition to the frequency of 100 kHz, frequencies of 100 ± 22 kHz are also observed in the frequency response of the sensor voltage, which are caused by nonlinear contact at the support location.

4. Conclusions

In this study, the effect of contact at the structure support on the nonlinear vibration behavior of the monitored structure was investigated by a combination of vibration and acoustics. In order to reduce the computational costs, which is an integral element of simulating high-frequency phenomena, the Fourier spectral element method was used, which has a great convergence rate compared to other similar numerical methods. Contact mechanics as a nonlinear factor governing the problem was introduced to the spectral element formulation using the penalty method. According to the results, in experimental and numerical tests, contact at the location of the defected support leads to the occurrence of modulated frequencies around the central band of the carrier frequency. The absence of modulated frequency bands in a pristine structure (baseline) allows the extraction of suitable damage-sensitive properties and early monitoring of the host structure.

5. References

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