

# Experimental and theoretical assessment of mode II fracture toughness for cracked ductile specimens with high strain-hardening

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## ABSTRACT

In this research, the mode II fracture toughness of O-notched diagonally loaded square plate samples with pre-existing cracks which are made of stainless steel 316L with specifications of highly ductile behavior and great strain hardening is investigated theoretically and experimentally. For this purpose, several fracture tests are carried out on the pre-cracked specimens to determine the fracture toughness experimentally. The experimental observations and the load-displacement curves obtained from the fracture tests illustrate that the pre-cracked specimens undergo large plastic deformations at the onset of crack propagation. Afterward, the fictitious material concept is used to estimate the values of fracture toughness achieved experimentally. By using fictitious material concept, the fracture toughness of pre-cracked specimens fabricated from stainless steel 316L could be estimated without the need for complicated and time-consuming elastic-plastic failure analysis and by performing only linear elastic analysis. For this purpose, the fictitious material concept is simply combined with mean stress, generalized mean stress, maximum tangential stress, generalized maximum tangential stress, strain energy density and generalized strain energy density criteria. It is shown that combination of fictitious material concept with four linear elastic brittle fracture criteria is quite successful in predicting the mode II fracture toughness of ductile pre-cracked specimens.

## KEYWORDS

Fictitious Material Concept, Crack propagation, Mode II loading, Elastic fracture mechanics, Fracture toughness.

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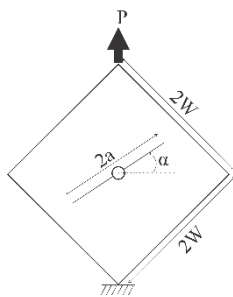
## 1. Introduction

Designers tend to use ductile materials in fabrication of engineering structures, since crack growth in such materials usually occurs gradually, which makes the periodic inspections of ductile members easier. Almost the majority of the research works on investigating ductile rupture have been all carried out based on elastoplastic analysis [1, 2]. In 2019, Torabi and Kamyab [3] introduced the Fictitious Material Concept (FMC). The load-carrying capacity (LCC) of specimens fabricated from materials with highly ductile behavior and great strain hardening could be estimated using this concept without the need for complicated and time-consuming elastic-plastic failure analysis.

The purpose of this research is to use the FMC to estimate mode II fracture toughness of O-notched diagonally loaded square plate (DLSP) samples with pre-existing cracks which are made of stainless steel 316L with specifications of highly ductile behavior and great strain hardening. For this purpose, several fracture tests are carried out on the DLSP specimens to determine the LCCs experimentally. Afterward, the FMC is used to estimate the values of LCCs achieved experimentally.

## 2. Experiments

As mentioned above, several fracture tests are carried out on the DLSP specimens to determine the LCCs experimentally. To make sure the results are reproducible and accurate, fracture tests are replicated three times for each DLSP specimen with a specific crack inclination angle. Figure 1 represents a schematic of the pre-cracked specimen.



**Figure 1. The pre-cracked specimen with the boundary and loading conditions and the geometrical parameters.**

The experimental observations and the load-displacement curves obtained from the fracture tests illustrate that the pre-cracked specimens undergo large plastic deformations at the onset of crack propagation.

## 3. Methodology

### 3.1. The Fictitious Material Concept

The LCCs of the DLSP specimens fabricated from stainless steel 316L could be estimated by using FMC without the need for complicated and time-consuming elastic-plastic failure analysis and by performing only linear elastic analysis. In fact, the FMC makes it possible to replace a virtual brittle material having linear elastic behavior with the stainless steel 316L with highly ductile behavior and great strain-hardening. Afterward, combining FMC with brittle fracture criteria enables the estimation of LCCs of the cracked DLSP samples.

The fictitious material concept needs two essential parameters, namely the virtual fracture toughness  $K_C^{FMC}$  and the virtual tensile strength  $\delta_f^{FMC}$ . It is assumed that the absorbed strain energy density (SED) of the real ductile material before necking is equal to the SED of the virtual brittle material needed for abrupt fracture. Moreover, the ultimate strain of the real ductile material is also considered to be equal to the fracture strain of the virtual brittle material. Considering the mentioned assumptions, the values of  $K_C^{FMC}$  and  $\delta_f^{FMC}$  are calculated equal to  $66.3 \text{ MPa}\sqrt{\text{m}}$  and  $1372 \text{ MPa}$ , respectively.

### 3.2. Linear elastic fracture criteria

In order to estimate the fracture toughness of the pre-cracked specimens made of stainless steel 316L theoretically, the FMC should be combined with mean stress (MS), generalized mean stress (GMS), maximum tangential stress (MTS), generalized maximum tangential stress (GMTS), strain energy density (SED) and generalized strain energy density (GSED) criteria. Therefore, in the governing equations of linear elastic fracture criteria,  $K_C^{FMC}$  and  $\delta_f^{FMC}$  are used instead of the plane strain fracture toughness ( $K_{Ic}$ ) and the material critical stress ( $\sigma_c$ ), respectively.

Equations 1 and 2 represent the governing equations of the two combined criteria FMC-GMS and FMC-GSED, respectively. Also, by assuming the T-stress equal to zero, the governing equations of FMC-MS and FMC-SED can be achieved.

$$K_c^{FMC} = \cos\left(\frac{\theta_0}{2}\right) \left[ -\frac{3}{2} K_{Ic} \sin(\theta_0) \right] + \frac{\sqrt{2\pi d_c^{FMC}}}{2} T \sin^2(\theta_0) \quad (1)$$

$$\left[ K_{Ic} (3 \cos(\theta_0) - 1) \right] - \frac{16}{3} T \sqrt{\frac{d_c^{FMC}}{2}} \sin\left(\frac{\theta_0}{2}\right) \cos(\theta_0) = 0$$

$$(K_c^{FMC})^2 = \frac{8\pi\mu}{(k-1)}(A_2 K_{IIc}^2 + A_3 T \sqrt{2r_c^{FMC}} \pi K_{IIc} + A_6 (T \sqrt{2r_c^{FMC}} \pi)^2) \\ C_2 K_{IIc}^2 + C_3 T \sqrt{2\pi r_c^{FMC}} K_{IIc} = 0 \quad (2)$$

The governing equations of MTS and GMTS criteria are extracted from [4] and [5], respectively. Therefore, the governing equations of the two combined criteria FMC-GMTS and FMC-MTS can be extracted as mentioned above, which are not repeated herein for the sake of brevity.

#### 4. Results and discussion

The experimental values of the fracture initiation angle ( $\theta_0$ ) and those of the ratio  $K_{IIc}/K_C^{FMC}$  are estimated using 6 combined criteria. Table 1 represents the theoretical and mean experimental values of  $K_{IIc}/K_C^{FMC}$  including the percent discrepancies. Also, Table 2 reports the theoretical and mean experimental values of fracture initiation angle ( $\theta_0$ ) under pure mode II loading including the percent discrepancies.

**Table 1. Theoretical and mean experimental values of  $K_{IIc}/K_C^{FMC}$  including the percent discrepancies ( $\Delta$ ).**

| Criterion/Experiment | $K_{IIc}/K_C^{FMC}$ | $\Delta$ (%) |
|----------------------|---------------------|--------------|
| Exp.                 | 0.87                | 0            |
| FMC-GSED             | 1.04                | 19.5         |
| FMC-SED              | 1.04                | 19.5         |
| FMC-GMS              | 0.79                | 9.2          |
| FMC-MS               | 0.86                | 1.1          |
| FMC-GMTS             | 0.79 [5]            | 9.2 [5]      |
| FMC-MTS              | 0.86 [4]            | 1.1 [4]      |

**Table 2. Theoretical and mean experimental values of fracture initiation angle under pure mode II loading including the percent discrepancies ( $\Delta$ ).**

| Criterion/Experiment | $\theta_0$ (Degrees) | $\Delta$ (%) |
|----------------------|----------------------|--------------|
| Exp                  | -73                  | 0            |
| FMC-GSED             | -80.1                | 9.7          |
| FMC-SED              | -79.6                | 9            |
| FMC-GMS              | -72.7                | 0.4          |
| FMC-MS               | -70.5                | 3.4          |
| FMC-GMTS             | -72.7 [5]            | 0.4 [5]      |
| FMC-MTS              | -70.5 [4]            | 3.4 [4]      |

Table 1 illustrates that the average discrepancies for FMC-GSED, FMC-SED, FMC-GMS, FMC-MS, FMC-

GMTS and FMC-MTS criteria are about 19.5%, 19.5%, 9.2%, 1.1%, 9.2% and 1.1, respectively.

According to Table 2, the average discrepancies for FMC-GSED, FMC-SED, FMC-GMS, FMC-MS, FMC-GMTS and FMC-MTS criteria are about 9.7%, 9%, 0.4%, 3.4%, 0.4% and 3.4, respectively. The comparison of experimental and theoretical results qualitatively reveals that the FMC-MS and FMC-GMS criteria can accurately estimate the fracture toughness and fracture initiation angle of pre-cracked specimens under pure mode II loading.

#### 5. Conclusions

- In this research, the governing equations of the mean stress criterion including the effects of T-stress were presented for the cracked specimens.
- By using fictitious material concept, the mode II fracture toughness of pre-cracked specimens fabricated from stainless steel 316L could be estimated without the need for complicated and time-consuming elastic-plastic failure analysis and by performing only linear elastic analysis.
- The comparison of experimental and theoretical results qualitatively revealed that the FMC-MS and FMC-GMS criteria can accurately estimate the fracture toughness and fracture initiation angle of pre-cracked specimens under pure mode II loading.

#### References

- [1] P. Kuhn, G. Catalanotti, J. Xavier, P. Camanho, H. Koerber, Fracture toughness and crack resistance curves for fiber compressive failure mode in polymer composites under high rate loading, *Composite Structures*, 182 (2017) 164-175.
- [2] F. Antunes, R. Branco, P. Prates, L. Borrego, Fatigue crack growth modelling based on CTOD for the 7050-T6 alloy, *Fatigue & Fracture of Engineering Materials & Structures*, 40(8) (2017) 1309-1320.
- [3] A.R.Torabi, M.Kamyab, The fictitious material concept, *Engineering Fracture Mechanics*, 209 (2019) 17-31.
- [4] F. Erdogan, G. Sih, On the crack extension in plates under plane loading and transverse shear, *J. Basic Eng*, 85(4) (1963) 519-525.
- [5] D.J.Smith, M.R.Ayatollahi, M.J.Pavier, The role of T-stress in brittle fracture for linear elastic materials under mixed-mode loading, *Fatigue & Fracture of Engineering Materials & Structures*, 24(2) (2001) 137-150.