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Study of the Analytical Solutions and Modelling of Nonlinear Buckling in Bellows

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ABSTRACT: Bellows are one of the widely used elements in various industries. These elements often contain an internal fluid flow and they are used as the length compensator, vibration absorber, and sealer. Usually, they are under pressure and out of range increasing in pressure can cause buckling of bellows. There are two types of buckling in bellows known as column buckling and in-plane buckling. In this paper first, a comprehensive study was done on analytical solutions of bellows buckling in the literature and then both types of buckling were investigated by using finite element method software ABAQUS and the results were compared with the experimental results that had been reported in references. According to the results, although elastic analysis of bellows buckling gives correct mode shapes, the buckling pressure is several times as much the experimental one. Then, buckling pressure is computed by applying imperfection according to the primary buckling mode shapes and nonlinear analysis was done by the use of the pressure-displacement curve. Finally, the necessary number of buckling mode shapes applied as imperfection and the amount of the mode shape amplitude required was investigated by comparing the results with the experimental one and finally good agreement with experimental results was obtained.

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1. INTRODUCTION

Excessive internal pressure may cause a bellows to become unstable and buckle. The two most common forms are column buckling and in-plane buckling. Column buckling is defined as a gross lateral shift of the center section of the bellows and in-plane buckling is characterized by tilting or warping of one or more convolutions. Column buckling is most associated with bellows which have a relatively large length-to-diameter ratio and in-plane buckling is most associated with bellows which have a relatively small ratio.[1]

Expansion Joint Manufacturers Association (EJMA) provides equations for each of these two types of instability. Tsukimori and Iwata [2] investigate types of bellows buckling by performing a number of experiments. Belyaev et al. [3] present the analytical relation for bellows buckling and investigate the accuracy of their analysis by using linear Finite Element Method (FEM) with ANSYS software. Williams [4] analyzed the bellows buckling using the nonlinear pressure-displacement method and compared the result with the equations in the papers and standards and reported the differences in FEM results with the EJMA's equations results. But neither of the two recent articles has compared their results with the experimental results.

In this paper, the analytical equations of both types of buckling are investigated. Then, to analyze them in two different bellows, linear FEM modelling is performed and its results are compared with the experimental results of Tsukimori and Iwata and EJMA equations. Then, by applying the initial imperfections and using nonlinear FEM (Riks method), the buckling pressures are obtained in accordance with the experimental results of Tsukimori and Iwata.

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2. METHODOLOGY

The column bellows buckling equations are developed based on the Euler buckling equation (Eq. (1)) and the correction coefficients obtained from the experimental results in the bellows design standards. In Eq. (1), F_{cr} is axial buckling force, E is the modulus of elasticity, I surface moment of inertia and L is the effective length. According to EJMA -the most comprehensive standard of bellows- column buckling pressure can be obtained from Eq. (2) with respect to the safety factor 2.25 where P_{sc} is column buckling pressure, N is the number of convolutions, F_{iu} is the axial spring rate per convolution and q is the convolution pitch. However, the in-plane buckling equations are presented based on the relationship between the internal pressure and the yield stress of the bellows material. In EJMA, in-plane buckling pressure is given by Eq. (3) with respect to the safety factor 1.75 where P_{si} is in-plane buckling pressure, A_c is the effective area, S_y^* is the corrected yield stress, D_m is the mean diameter, α is the in-plane instability stress interaction factor and k_r is the circumferential stress factor.

$$F_{cr} = \frac{4\pi^2 EI}{L^2} \tag{1}$$

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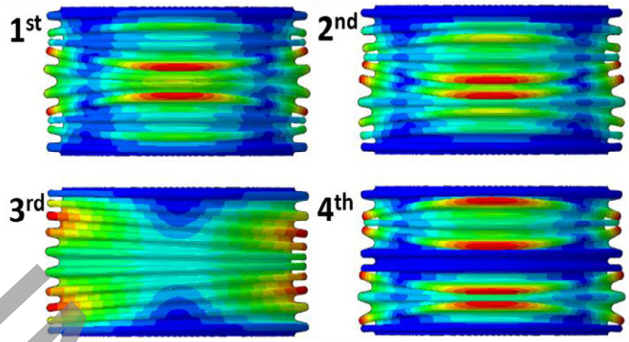

Fig.. 1. 4 modes of buckling in 10 convolution bellows

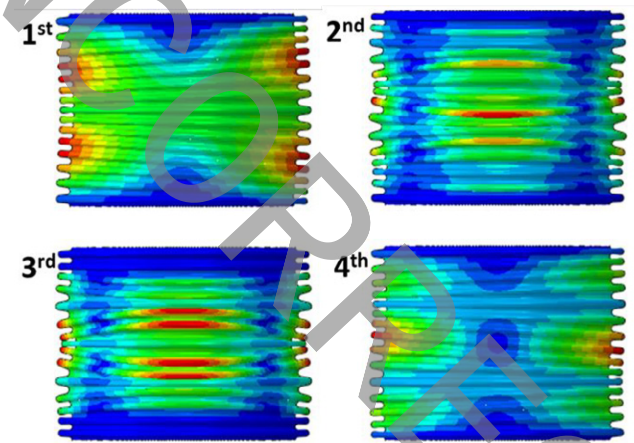

Fig. 2. Four modes of buckling in 15 convolutions bellows

$$P_{sc} = \frac{0.34\pi f_{iu}}{N^2 q} \tag{2}$$

$$P_{si} = \frac{1.3 A_c S_y^*}{K_r D_m q \sqrt{\alpha}} \tag{3}$$

Linear buckling FEM is performed for two bellows, which have been already analyzed by Tsukimori and Iwata, using the eigenvalue method in ABAQUS software. The results are then compared with the values obtained from EJMA equations and the experimental results of Tsukimori and Iwata [2]. It is observed that the results are not enough accurate. In nonlinear FEM modelling, the results depend on mesh and element size in addition, and bellows only swelled and entered the plastic deformation but not buckled. Accordingly, the only way is using initial imperfection. In order to apply the imperfection to bellows, the buckling mode shapes of this structure are derived from linear analysis. There are two variables to apply imperfection. The first variable is the number of buckling mode shapes and the second is the coefficient of action for each buckle shape mode. In order to investigate the effect of these two variables, in both bellows, the number of different modes with different displacement coefficients is applied to the initial model and the results of the nonlinear analysis are compared.

Table 1. Buckling pressure from various approaches

N	Buckling pressure (MPa)				
	EXP [2]	EJMA [1]		FEM	Eq (1)
		In-plane	column		
10	0.836	0.646	5.978	4.094	-
15	0.707	0.646	0.465	2.535	2.656

3. RESULTS AND DISCUSSION

Figs. 1 and 2 show the first four buckling mode shapes for bellows with ten and fifteen convolutions resulting from linear FE analysis. In the ten convolutions bellows the first, second, and fourth modes are in-plane buckling and the third mode is column buckling. But in the bellows with fifteen convolutions, the first and fourth modes are column buckling, and the second and third modes are in-plane buckling. As expected, first buckle mode in a relatively small length-to-diameter ratio is the in-plane buckle, and in a relatively large length-to-diameter ratio is the column buckle.

Table 1 compares the buckling pressure results obtained from linear FE analysis with experimental, EJMA and Euler results. According to EJMA, the first buckling mode in ten convolutions bellows is in-plane buckling and in fifteen convolutions bellows is column buckling, which is in agreement to the linear FEM results (Figs. 1 and 2). In fifteen convolutions bellows, there is a good agreement between the Euler buckling equation (Eq. (1)) and the linear FEM result. But the most important result of Table 1 is the large difference between the FEM results and the experimental results in spite of the correct buckling type. There is a very clear reason for this difference. The linear FEM considers full bellows geometry without any defects, while real bellows can not be a complete cylinder and have quite identical congresses, and these are the defects that cause the bellows to buckle much earlier. Therefore, it is necessary to incorporate these deficiencies into FEM, which requires nonlinear analysis.

In the nonlinear FEM for ten-convolution bellows, the effect of applying 1 to 10 number of buckling mode shapes with a maximum 10% of bellows thickness as the imperfection on the buckling pressure value is investigated. The result is shown in Fig. 3. Accordingly, the magnitude of the difference in buckling pressure by applying one mode versus ten modes is only 0.5%. Then, the effect of applying imperfection with values ranging from 5% to 100% bellows thickness is investigated. The result is shown in Fig. 4. Accordingly, the difference is only 6.36%. It should be noted that the difference between the results of the linear FEM with the nonlinear FEM by applying one mode shape with 10% of thickness as imperfection is equal to 402%.

Since the effect of the number of buckling modes as an imperfection was negligible, the effect of applying buckling modes as the imperfection with the values ranging from 5% to 100% of bellows thickness was investigated in fifteen-convolution bellows (Fig. 5). Accordingly, the difference in model with a maximum of 5% versus a 100% bellows thickness is only 7.76%. But the difference between the results of the

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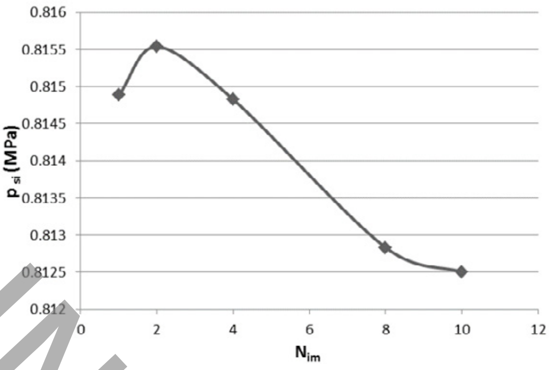

Fig. 3. P_{si} - N_{im} in 10 convolution bellows

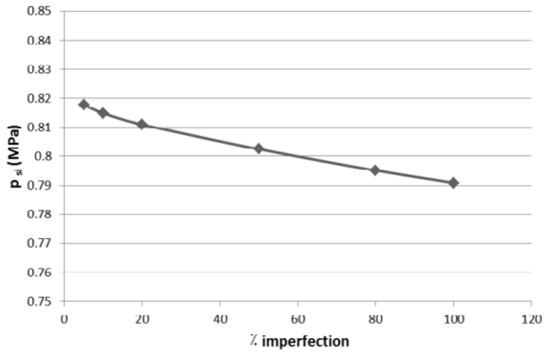

Fig. 4. P_{si} - % imperfection in 10 convolution bellows

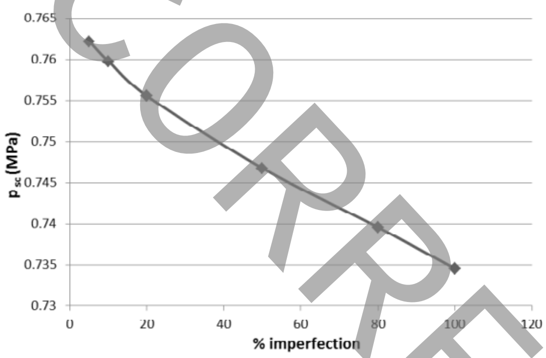

Fig. 5. P_{sc} - % imperfection in 15 convolution bellows

Table 2. Buckling pressure from various FEM approaches

N	Buckling pressure (MPa)				%Error
	EXP [2]	EJMA [1]	Eigen value linear FEM	Nonlinear imperfection FEM	
10	0.836	0.646	4.094	0.816	2.4
15	0.707	0.465	2.535	0.759	7.3

linear FEM with the nonlinear FEM by applying one mode shape with 10% of thickness as imperfection is equal to 234%.

Table 2 presents the buckling pressure results obtained from nonlinear FEM (with one mode and 10% imperfection), linear FEM, experimental results and EJMA formulas. Accordingly, using of imperfection and nonlinear FEM has a great improvement in the results and cause a very high accuracy in predicting the bellows buckling pressure. Since the results of this method are compared with the experimental results and are in very good agreement (Table 2), this method of FE analysis is preferred over the linear method of Belyaev et al. [3] and nonlinear method of Williams [4] both of which did not compare their results with experiments.

4. CONCLUSIONS

In the case of FE analysis of bellows buckling the following results can be stated:

• Linear FE analysis of bellows buckling is not very efficient and provides critical buckling pressure far above the actual value but provides the type and shape of the buckling modes correctly.

• Using nonlinear FEM by applying buckling mode shapes obtained from linear analysis as the initial imperfection, provide accurate results compared to the experimental results (with a maximum error of about 5%).

• The effect of imperfection in nonlinear FEM on buckling pressure is very high and must be considered. But its value and the number of mode shapes applied as an imperfection have little effect on the results.

• Standard equations are always conservative in design applications compared to experimental and nonlinear FEM results.

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