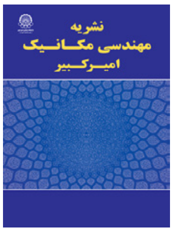

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Investigation of Quality Factor for Linear Vibrations of Rectangular Micro-plates with Thermoelastic Damping

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ABSTRACT: Regarding the necessity of obtaining high-quality resonators in micro-electromechanical systems, recognizing and investigating the parameters that affect the quality factor of micro-structures are essential and inevitable. Thermoelastic damping is a dominant source of damping which has a considerable effect on the quality factor. In micro-electromechanical systems, microplates are used as resonators and radio frequency filters and so on. In this paper, the effect of thermoelastic damping, which is one of the most important factors affecting the quality factor, has been investigated for rectangular micro-plates. The micro-plate is subjected to an electrostatic actuation. Galerkin method has been used to simplify and solve the governing equations. The result is a nonlinear algebraic equation for the quality factors of microplates of general conditions due to thermoelastic damping. Unlike previous researches, the proposed model can directly calculate the quality factor and there is no need of calculating undamped natural frequency. COMSOL multiphysics software is used for finite element simulation. After verification of the proposed model, the effect of various parameters on the quality factor is investigated. The proposed model can also be used to calculate the pull-in instability voltage. The results of the current paper can be used to design micro-electromechanical systems.

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1- INTRODUCTION

In recent years Micro-Electromechanical Systems (MEMS) have gained the attention of many researchers due to their unique features such as light weight, small dimensions and low energy consumption [1]. Micro-plates are used in many applications, such as resonant sensors and RF filters [2]. For these applications, it is necessary to consider the effects of damping as it can affect the quality factor of the micro-structure. There are many damping mechanisms that contribute to lowering the quality factors of microstructures. Thermoelastic damping has a significant effect on the performance of micro-resonators because it determines the quality factor of the resonators [3, 4]. When a structure vibrates in a vibrational mode there are some regions of compression and some of the extension. Depending on the timescale of the vibration heat flows from the warmer parts of the micro-structure to the cooler parts. Since heat flow is an irreversible process, this heat flow is associated with energy loss from the vibrational mode and corresponding damping for the resonant mode [5].

Motivated by the aforementioned remarks, in this paper the quality factor of rectangular micro-plates for the cases of free and forced vibrations is investigated. Galerkin decomposition method is used to discretize the governing equations. Finite element simulation is also carried out using COMSOL multiphysics software. It is shown that the FE simulation is reliable. The results of the present work can be used to design MEMS resonators.

2- MODELING

Here we consider a rectangular micro-plate as Fig. 1 subjected to an electrostatic load.

Assuming small strains and displacements, we obtain the following thermoelastic linear equation of motion as [6]:

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} + N^{\mathrm{T}} \nabla^2 w + \nabla^2 M^{\mathrm{T}} = \frac{\varepsilon_0 V_{\mathrm{DC}}^2}{d^3} w \tag{1}$$

where $w(x, y, t)$ is the transverse deflection of the plate at the position x and y at time t, ρ is the material density, d is the initial gap width, $D = Eh^3/12(1-v^2)$ is the plate flexural rigidity, h is the micro-plate thickness, E is Young's modulus, v is Poisson's ratio and M^{T} is the thermal bending moment and is defined as

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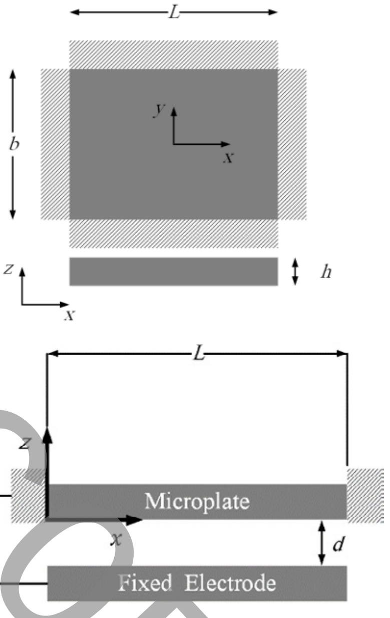

Fig. 1. Schematic model of the micro-plate

$$M^T = \frac{E\alpha_t}{1-\nu}\int_{-\frac{h}{2}}^{\frac{h}{2}} z\,\theta dz \qquad (2)$$

where $\theta = T - T_0$. The temperature distribution is governed by the classical heat conduction equation as:

$$K\nabla^2\theta + q = \rho C_P\frac{\partial\theta}{\partial t} - \frac{E\alpha_t T_0}{1-\upsilon}\frac{\partial}{\partial t}\left(z\nabla^2 w\right) \qquad (3)$$

where K is the thermal conductivity and Cp is the heat capacity coefficient at constant pressure. For the linear damped eigenvalue problem, we have:

$$\begin{aligned} w(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varphi_{mn}(x,y)e^{i\Omega_{mn}t} \\ \theta(x,y,z,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Theta_{mn}(x,y,z)e^{i\Omega_{mn}t} \end{aligned} \qquad (4)$$

where $\varphi_{mn}(x, y)$ and $\theta_{mn}(x, y, z)$ are the nth complex mode shapes of the plate and the associated temperature variation, respectively and Ω_{mn} is the nth complex eigenvalue. Based on the Galerkin method the following equations can be obtained.

$$\left(D+\tilde{D}\right)\left(a_1+a_2+a_3\right) - \left(\rho h\Omega_{11}^2 + \frac{\varepsilon_0 V_{DC}^2}{d^3}\right)a_4 = 0 \qquad (5)$$

where:

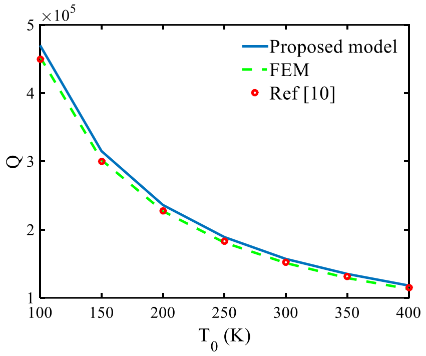

Fig. 2. Variation of the quality factor of the first mode with the temperature

$$\tilde{D} = \frac{E^2\alpha^2 T_0}{(1-\upsilon)^2\rho C_P}\left(\frac{h^3}{12}+\frac{h}{K_P^2}-\frac{2\tan\left(\frac{K_P h}{2}\right)}{K_P^3}\right) \qquad (6)$$

$$K_P = (1-i)\sqrt{\frac{\Omega_{mn}\rho C_p}{2K}}$$

and:

$$\begin{aligned} a_1 &= \iint_A \frac{\partial^4\varphi_{11}}{\partial x^4}\varphi_{11}dxdy \\ a_2 &= \iint_A \frac{\partial^4\varphi_{11}}{\partial x^2\partial y^2}\varphi_{11}dxdy \\ a_3 &= \iint_A \frac{\partial^4\varphi_{11}}{\partial y^4}\varphi_{11}dxdy \\ a_4 &= \iint_A \varphi_{11}^2 dxdy \end{aligned} \qquad (7)$$

The quality factor is defined as:

$$Q = \frac{1}{2}\left|\frac{\mathrm{Re}(\Omega_{11})}{\mathrm{Im}(\Omega_{11})}\right| \qquad (8)$$

3- RESULTS AND DISCUSSION

In this section, as a case study, we present the numerical results for the quality factor of a fully clamped silicon micro-plate. Table 1 lists the dimensions of the micro-plate. Fig. 2 shows the variation of Q of the first mode of a plate with T_0. It is obvious that there is a very good agreement between the results of the present study and Ref. [6].

Fig. 3 shows the effect of applied DC voltage on the quality factor. It can be seen that as the voltage increases the quality factor decreases.

4- CONCLUSIONS

In this study, we derived a semi-analytical expression for the quality factor of rectangular micro-plates of general boundary conditions. Comparing the calculated quality factors using the proposed semi-analytical model and FE

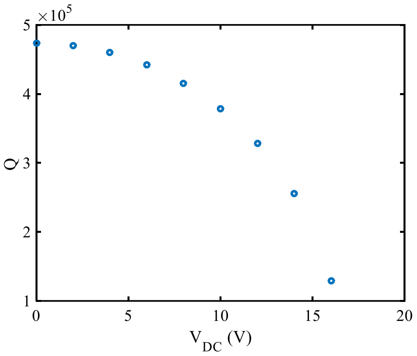

Fig. 3. Variation of the quality factor of the first mode with the DC voltage

simulation to those of in Ref. [6] verifies our calculations.

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Table 1. Dimensions of the micro-plate (μm)

L	200
b	100
h	1.5

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