



## Transient Analysis and Free Vibration of Functionally Graded Truncated Conical Shells Subjected to Moving Pressure

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**ABSTRACT:** In this paper, an efficient and accurate solution method is developed for transient analysis and free vibration of functionally graded truncated conical shell, subjected to symmetric internal or external moving pressure. The material properties of the shell are graded continuously in the radial direction according to a Mori-Tanaka and volume fraction power-law distribution. A hybrid solution method composed of the layerwise theory, differential quadrature method and Fourier series expansion is employed to investigate the aforementioned problem. A Fourier series expansion is used for the displacement components and dynamic pressure in the axial direction. Then the layerwise theory across the thickness direction in conjunction with Hamilton's principle is employed to obtain equations of motion and boundary conditions. Eventually, the differential quadrature method is implemented to discretize the governing equations in the time domain. This research shows some interesting results that can be helpful for the design of functionally graded shells subjected to moving pressure. The developed results are successfully compared with the available results in the literature. The convergence study demonstrates the fast convergence rate with a relatively low computational cost. The results reveal that a free vibration with significant amplitude is generated due to excitation from the transition of the moving pressure.

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## 1. INTRODUCTION

The considerable advantages of Functionally Graded Materials (FGMs) over conventional materials and increased usage of the shells in engineering applications have stimulated considerable interest in the dynamic analysis of such shells. Few studies have been done in the field of axisymmetric Functionally Graded (FG) shells subjected to moving pressure and most of them are restricted to the cylindrical hollow shells.

Therefore the importance of this problem motivated us to study the dynamic behavior of FG truncated conical shells subjected to axisymmetric moving pressure. In this paper, the free vibration caused by excitation of the moving load is also be considered. Since the exact solution is not available for the described problem, the objective of the presented investigation is to provide an efficient and accurate hybrid method based on layerwise theory, Differential Quadrature Method (DQM) and Fourier series expansion.

## 2. METHODOLOGY

Consider a thick functionally graded truncated conical shell with varying material properties in the thickness

direction. The shell is subjected to local symmetric moving pressure  $p(z, t)$  with length  $l_s$  and axial velocity of  $V$ . The inner and outer radius of the shell are denoted by  $r_{in}$  and  $r_{out}$ , respectively,  $l$  is the length of the cone and  $\psi$  is the semi-vertex cone angle. Also  $u$ ,  $v$  and  $w$  are the radial, circumferential and axial displacement components, respectively. The distribution of mechanical properties in the radial direction of the FG shell are assumed to obey the Mori-Tanaka model. The density  $\rho$  of the FG shell is defined by using a simple power-law distribution of volume fraction.

The displacement field of the truncated conical shell subjected to axisymmetric moving load is expressed by using the Fourier series in the axial direction. Then Reddy's layerwise theory [1] is used. Based on the layerwise theory the thickness of the FG shell is divided into  $N_r - 1$  computational layers and the axisymmetric displacement components  $U_m$  and  $W_m$  are expressed in accordance with Eq. (1) [2] as follows:

$$\begin{aligned} u(r, z, t) &= U_m(t) \varphi_i(r(z)) \cos(\lambda_m z) \\ w(r, z, t) &= W_m(z, t) \varphi_i(r(z)) \sin(\lambda_m z) \end{aligned} \quad (1)$$

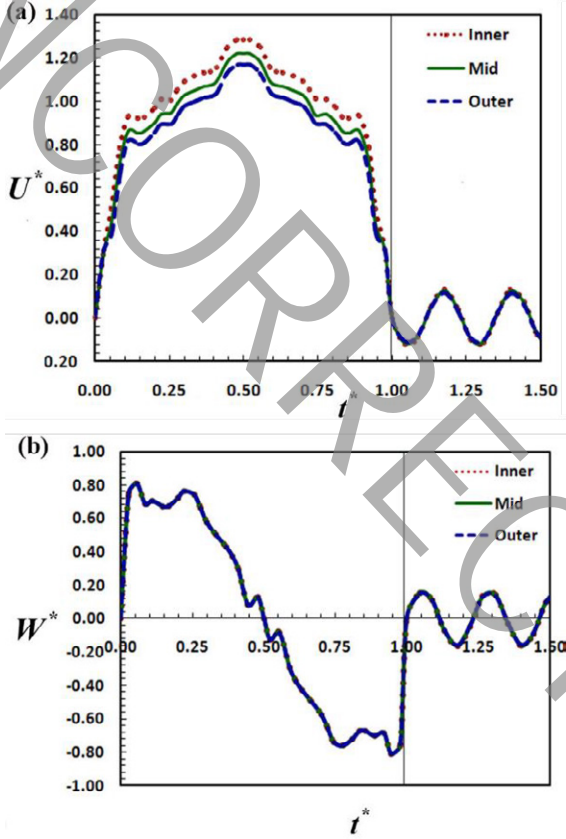
The truncated conical shell can be subjected to either

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**Table 1. Geometric characteristic and material properties of the FG truncated cone**

Material properties					
$K_f$ (GPa)	$K_c$ (GPa)	$G_f$ (GPa)	$G_c$ (GPa)	$\rho_f$ (kg/m <sup>3</sup> )	$\rho_c$ (kg/m <sup>3</sup> )
58.3	125.8	26.9	58.1	2707	3000
Geometric characteristic					
$L$ (m)	$r_{in}$ (m)	$r_{out}$ (m)			
1	$0.05+0.03x$	$0.07+0.03x$			

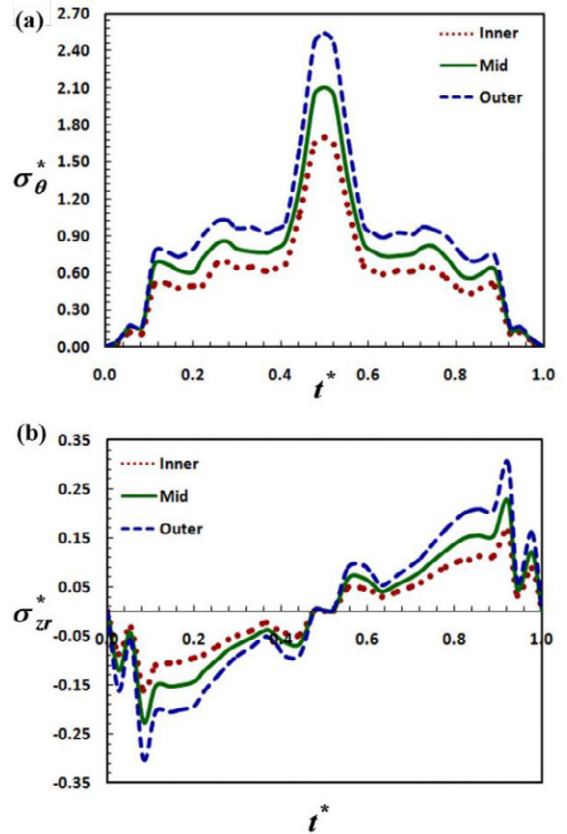


**Fig. 1. Time histories of the dimensionless a) radial and b) axial displacements at the mid-length of the FG shell**

internal moving pressure  $p_{in}(t, z)$  or external moving pressure  $p_{out}(t, z)$  as Eq. (2):

$$\begin{aligned}
 p_{in}(z, t) &= \bar{p}_{in}(t) \{u_s[z - z_1(t)] - u_s[z - z_2(t)]\} \\
 p_{out}(z, t) &= \bar{p}_{out}(t) \{u_s[z - z_1(t)] - u_s[z - z_2(t)]\}
 \end{aligned}
 \tag{2}$$

Hamilton's principle is employed to obtain the governing equations of motion. To solve the equations of motions, the differential quadrature method is used. According to the DQM, the time derivatives of displacement components at the  $i^{th}$  computational layer is approximated by a weighted linear summation of the displacement components. By solving the governing equations of motions for each axial wave number, the time histories of the axial and radial displacement



**Fig. 2. Time histories of the dimensionless a) tangential and b) shear stresses at the mid-length of the FG shell**

components in each point of the FG truncated conical shell are predicted. To develop the governing equations for the case of free vibration, the dynamic pressures are set to zero.

### 3. DISCUSSION AND RESULTS

The constituent materials of the FG truncated cone is composed of aluminum and zirconia ceramic and vary smoothly from aluminum metal at the inner surface to zirconia ceramic at the outer surface. In Table 1, the geometric characteristic and material properties of the truncated cone are presented. Here, the inner surface of the shell is subjected to symmetric moving pressure that is related to Eq. (3).

$$p_{in}(t, z) = \{u_s[z - 100t] - u_s[z - 100(t + 0.001)]\} \tag{3}$$

Fig. 1 illustrated the time histories of dimensionless radial and axial displacements on the inner, mid and outer surfaces of the FG truncated conical shell at its mid-length. It is observed that the free vibration amplitude is relatively significant. This is arising from considerable high radial and axial displacement rates due to excitation from the transition of moving pressure.

Time histories of the dimensionless tangential, and shear ( $\sigma_x^*$ ) stress components on the inner, mid and outer surfaces at the mid-length of the FG shell are performed in Fig. 2. It is observed that the outer surface has to endure more stresses due to its higher stiffness. This is while that, by replacement

**Table 2. Effects of moving pressure area on maximum magnitudes of  $U^*$  and  $\sigma_\theta^*$** 

$l/L$	$P=\text{const.}$		$F=\text{const.}$	
	$U^*$	$\sigma_\theta^*$	$U^*$	$\sigma_\theta^*$
0.07	1.0583	1.0839	1.5118	1.5484
0.10	1.2853	1.4958	1.2853	1.4958
0.13	1.5904	1.7478	1.2234	1.3445
0.30	3.0348	2.4460	1.0116	0.8153
0.41	4.0524	3.0460	0.9884	0.7429
0.42	4.0924	3.1225	0.9744	0.7435
0.45	4.3711	3.5129	0.9714	0.7806
0.48	4.8138	3.9455	1.0029	0.8220
0.52	5.1574	4.4500	0.9918	0.8558
0.56	5.6721	4.7188	1.0129	0.8426
0.58	5.7954	5.3233	0.9992	0.9178

of the FG shell with a homogenous and isotropic shell, the maximum stresses will occur on the inner surface.

Table 2, listed maximum magnitudes of  $U^*$  and  $\sigma_\theta^*$  on the inner surface at the middle of the cone length, for two cases of constant pressure ( $P=\text{const.}$ ) and constant load magnitude ( $F=37.7$  kN). It is seen that, in the case of  $P=\text{const.}$ , by increasing the moving pressure area, the amount of radial displacement and tangential stress will also increase. However, in the case of constant load magnitude, increasing the area of moving pressure leads to a decrease in the magnitude of applied pressure.

#### 4. CONCLUSIONS

In this paper, an accurate numerical hybrid method based on the layerwise theory, Fourier series expansion and DQM is presented. By employing this method, the transient behavior and free vibration response of FG truncated conical shells subjected to moving pressure are investigated. At first, a

Fourier series expansion is employed to express the dynamic pressure and displacements in the axial direction. Then the layerwise theory in conjunction with Hamilton's principle is implemented to approximate the equations of motion in the radial direction. Finally, DQM is used to discretize the governing equations in the time domain. Subsequently, by using this method, a FG truncated conical shell was modeled and the detailed responses of the shell subjected to a moving pressure are investigated.

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