



## Impact Time Guidance Law against Maneuvering Targets Using Sliding Mode Control

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**ABSTRACT:** Controlling the impact time of missiles to the target has great importance in applications such as the cooperative attack of multiple missiles since multiple missiles can be reached a specified point simultaneously. This paper presents a sliding mode based guidance law, to control the impact time against maneuvering targets. At first, by selecting an appropriate switching surface and using nonlinear engagement dynamics, a new sliding mode control has been designed which, the sufficient conditions of its stability are derived using the Lyapunov stability theorem. The sliding surface has been selected such that the line of sight rate and the error of the time to go converge to zero at the same time. By considering the nonlinear dynamic equations of maneuvering targets, this will guarantee to impact the target at the desired time. Unlike the other similar methods, our proposed method does not need the assumptions of small flight path angle and stationary target. Using an engagement simulation model, the effectiveness of the proposed method is shown for different scenarios (static and maneuvering targets) and different impact times. In the end, the comparison results with two similar methods are also presented.

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## 1. INTRODUCTION

The main purpose of missiles is to hit targets with zero miss. Proportional navigation guidance has been widely studied and utilized in light of the ease of implementation and proper operation. As the targets become more maneuverable and higher precision and destructive power of intercept missiles are demanded, the need for more advanced guidance rules is increasing. In particular, in recent years, anti-missile defense systems have also been developed, and limitations on conventional guidance techniques can lead to the successful destruction of the missiles.

In order to increase the likelihood of missile success, in addition to achieving zero miss, it is necessary to take into account other features such as final speed, final impact angle, and impact time. One of the characteristics of optimal guidance laws is that they need to estimate time to go ( $t_{go}$ ). Inaccurate estimation of this parameter not only severely damages the guidance performance, but also significantly diverts the overall path from the optimal one. Tuck et al. [1] have provided a method for updating time to go calculations, taking into account the direct deviation of the missile. Given the real condition of non-constant velocity, the optimal guidance law and  $t_{go}$  have been developed by Chow et al. [2]. Rayo et al. [3] have obtained an optimum guidance law with the final impact angle constraint.

In this paper, a new sliding mode guidance law is

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derived to control the impact time against maneuvering targets. Considering the nonlinear dynamic equations of the maneuverable target, the proposed controller will ensure that the missile reaches the maneuverable target at the desired impact time.

## 2. METHODOLOGY

In this section, the mathematical model of missile-target engagement is presented. The geometry of the missile-target planar engagement is shown in Fig. 1, where  $M$  and  $T$  represent the missile and target, respectively.

It is assumed that missile and target are moving at a constant velocity. Consider that  $V_M$  and  $V_T$  are the velocity,  $\varphi_M$  and  $\varphi_T$  are flight path angles,  $a_M$  and  $a_T$  are the acceleration of missile and target, respectively. Kinematic equation of missile-target planar engagement is depicted hereunder:

$$\begin{aligned} \dot{r} &= V_r = -V_M \cos(\theta_M) + V_T \cos(\theta_T) \\ r\dot{q} &= V_q = -V_M \sin(\theta_M) + V_T \sin(\theta_T) \\ \dot{\varphi}_M &= \frac{a_M}{V_M} \cdot \theta_M = \varphi_M - q \\ \dot{\varphi}_T &= \frac{a_T}{V_T} \cdot \theta_T = \varphi_T - q \end{aligned} \quad (1)$$

$r$  is the relative position of the missile and the target and



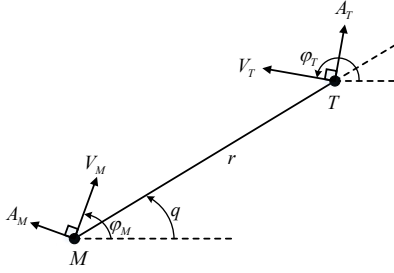


Fig. 1. Engagement geometry of missile-target.

$q$  is the line-of-sight angle. After some algebra:

$$\begin{aligned} \ddot{q} &= -\frac{2\dot{r}\dot{q}}{r} - \frac{\cos(\theta_M)}{r} a_M + \frac{\cos(\theta_T)}{r} a_T \ddot{r} \\ &= -a_T \sin(\theta_T) + a_M \sin(\theta_M) + r\dot{q}^2 \end{aligned} \quad (2)$$

In the missile-target engagement scenario, the impact time is defined as the sum of current time and time to go:

$$t_f = t + t_{go} \quad (3)$$

$t_{go}$  is variable and depends on future control inputs. If the missile guidance law is designed and specified along the entire path, the mathematical relationship of the time to go can be obtained, but it is impossible to calculate the exact value of this parameter. In order to solve this problem, most articles have used the expressions for time to go estimation instead of its unknown exact value. One of the most widely used estimated expression of  $t_{go}$  is:

$$t_{go} = -\frac{r}{\dot{r}} = -\frac{r}{V_r} \quad (4)$$

where  $V_r$  is the derivative of relative position.

There are several choices for the sliding surface. In this paper, similar to Ref. [4], the following relation is taken as the sliding surface:

$$s = \dot{q} + C e \operatorname{sgn}(\dot{q}) \quad (5)$$

where  $C$  is a positive constant and  $e$  is the impact time error. Selecting the above sliding surface can guarantee to achieve the desired goals by nullifying the line-of-sight rate, the first term, and the timing error of the second term. The time to go error can be written as:

$$e = t_f - t_f^d = t + t_{go} - t_f^d = t_{go} - t_{go}^d \quad (6)$$

where  $t_{go}^d$  is the estimation of time to go and are calculated according to Eq. (4).  $t_{go}^d = t_f^d - t$  indicates the desired amount of  $t_{go}$ . To design the time control law, by differentiating the sliding surface (Eq. (5)) and inserting differentiating the sliding surface (Eq. (5)) and by inserting  $\dot{e}$  from Eq. (6), we have:

$$\dot{s} = \ddot{q} + C(1 + \dot{t}_{go}) \operatorname{sgn}(\dot{q}) \quad (7)$$

By differentiating Eq. (4) and replacing it from Eq. (1), one has:

$$\begin{aligned} \dot{t}_{go} &= \frac{r\dot{V}_r}{V_r^2} - 1 = \frac{r}{V_r^2} r\dot{q}^2 + a_M \sin(\theta_M) - \\ a_T \sin(\theta_T) &= \frac{1}{V_r^2} (V_q^2 + a_M r \sin(\theta_M) - a_T r \sin(\theta_T)) \end{aligned} \quad (8)$$

Finally, after some algebra, the control signal is depicted hereunder:

$$\begin{aligned} a_M &= a_M^{eq} + a_M^{disc} \\ &= \frac{-2\dot{r}\dot{q}}{r} + \frac{\cos(\theta_T)}{r} a_T + \\ &= \frac{C \operatorname{sgn}(\dot{q})(V_q^2 - a_T r \sin(\theta_T))}{V_r^2} \\ a_M^{eq} &= \frac{\cos(\theta_M)}{r} - \frac{C \operatorname{sgn}(\dot{q}) r \sin(\theta_M)}{V_r^2} \\ a_M^{disc} &= K \operatorname{sgn}(s) = -\frac{M \operatorname{sgn}(s)}{\operatorname{sgn}\left(\frac{\cos(\theta_M)}{r} - \frac{C r \sin(\theta_M) \operatorname{sgn}(\dot{q})}{V_r^2}\right)} \end{aligned} \quad (9)$$

To overcome the high chattering input control due to  $\operatorname{sgn}(\cdot)$  function, it is proposed to use the sigmoid( $\cdot$ ) function instead:

$$K = \frac{M}{\operatorname{sgn}\left(\frac{\cos(\theta_M)}{r} - \frac{C r \sin(\theta_M) \operatorname{sigmoid}(\dot{q})}{V_r^2}\right)} \quad (10)$$

### 3. RESULTS AND DISCUSSION

In order to evaluate the performance of the proposed impact time control guidance, two scenarios with stationary and maneuverable targets have been considered. In order to show the efficiency and superiority of the proposed method, its performance has been compared with Ref. [5] and Ref. [6]. The simulation results are shown below. For the stationary target, the control input is applied for different desired impact times and the results are presented in Figs. 2 to 5. It can be seen in Fig. 2 that, by increasing the desired impact time, the missile trajectory moves further away from the target at the beginning of the flight. In fact, to get later to the target, more curvature is needed in the missile's flight path.

The results for maneuvering target are shown in Figs. 6 to 9. The simulation result of compared methods of Ref. [5] and Ref. [6] are shown in Figs. 7 and 6, respectively. Our proposed method result is demonstrated in Fig. 8. Simulations are performed until the desired impact time of 40 seconds. It can be clearly conceived that the desired impact time is only achieved in Fig. 8 (our proposed method) and the impact time of Figs. 6 and 7 are less and more than the desired one

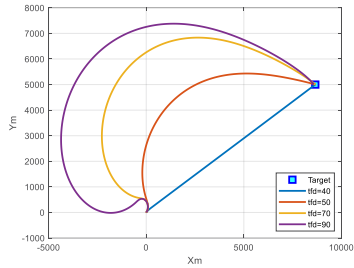


Fig. 2. Missile trajectory (stationary target).

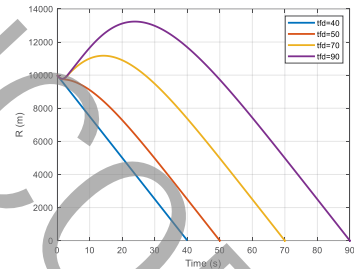


Fig. 3. Missile-target relative position.

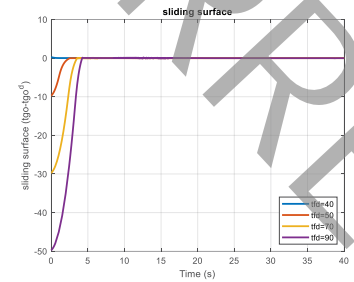


Fig. 4. Sliding surface.

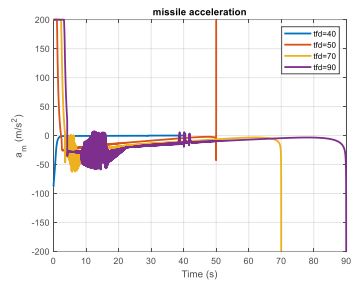


Fig. 5. Guidance command ( $a_M$ ).

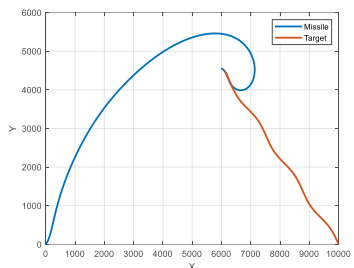


Fig. 6. Missile and maneuvering target trajectories [6].

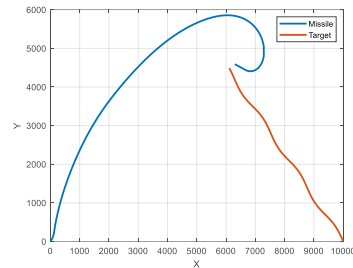


Fig. 7. Missile and maneuvering target trajectories [5].

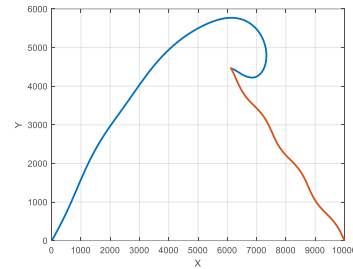


Fig. 8. Missile and maneuvering target trajectories (proposed method).

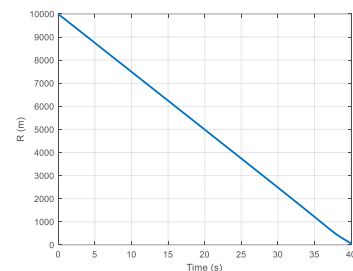


Fig. 9. Missile-target relative position.

#### 4. CONCLUSION

Due to the high importance of impact time control, in this study, a new sliding-mode based guidance law is proposed which by choosing a new sliding surface involving time to go error, and considering nonlinear relationships of missile and target engagement, can govern the impact time successfully. The effectiveness of the proposed impact time control was evaluated by simulation and it was shown that this new guidance law has also the capability of handling maneuvering targets at a predetermined impact time while the comparative methods have failed to reach the desired time.

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respectively. Fig. 9 shows the separating distance between the missile and target, which is decreasing monotonically.

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