

Sensitivity Analysis of Rotor Dynamic Behavior to Manufacturing Tolerances Based on Global Sensitivity Analysis and Statistical Methods

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ABSTRACT

Engineering structures are inevitably exposed to various sources of uncertainty. The uncertainty in the parameters led to structures with identical nominal parameters having different vibrational behavior, such as different natural frequencies. Therefore, it is inevitable to consider parameter variability for a robust design. The rotational motion of turbomachinery makes vibration an important issue in their design. Therefore, it is essential to accurately determine the vibrational behavior of rotating systems and the parameters affecting them. No comprehensive experimental study is reported on the sensitivity of vibration behavior of industrial rotating systems to parameter uncertainty in the related literature. Therefore, in this paper, a powerful method of global sensitivity analysis based on variance analysis is presented using an industrial compressor sample to determine the effective parameters in its response uncertainty. The Monte Carlo simulation method is adopted to implement the global sensitivity analysis method. In this method, the uncertainty in the system response quantifiably devotes itself to the uncertainty of its parameters and provides a quantitative analysis along with qualitative predictions to the designer. The presented method in this paper can be very useful in designing rotating machinery and identifying sensitive parameters on the system response for the codification of design and manufacturing instructions, like component tolerance.

KEYWORDS

Rotor dynamics, Global sensitivity analysis, Monte Carlo, Uncertainty, Model Updating.

Introduction

Increasing the accuracy of the finite element model is an important part of engineering community researches because a finite element model with the ability to estimate the exact prediction of system response can help engineers and researchers to accurately predict the behaviors of engineering structures in the real world, and provides more manageable and more accurate decision making

Due to the need to consider the parameter uncertainty to model the dynamics of structures accurately, the use of sensitivity analysis in these systems is inevitable. Sensitivity analysis assigns uncertainty in the output of a model to different sources of uncertainty in its input [1]. In general, sensitivity analysis can be divided into local sensitivity analysis and global sensitivity analysis[1, 2].

Local sensitivity analysis focuses on the local effects of factors. Local sensitivity analysis belongs to the class of one-factor methods at a time, in which one parameter is

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slightly altered at a time, while the rest are kept constant. On the other hand, global sensitivity analysis is to sensitivity analysis is a useful approach to quantify the effect of input uncertainty on the objective function and can be used to distinguish important factors from unimportant ones [2].

Various researchers [3, 4] have used the global sensitivity method to identify important parameters in various structural dynamics problems and concluded that the global sensitivity method is a reliable way to identify the effective parameters by considering the interaction between the parameters. Studies have shown that tolerances and uncertainties in input parameters affect the dynamic behavior of rotating systems [5, 6], but there is no comprehensive study for sensitivity analysis in the rotor dynamics field.

In this paper, a suitable framework in the sensitivity analysis of rotating systems based on variance decomposition and the Monte Carlo method is presented. It provides quantitative analysis and qualitative predictions, which is useful in designing rotating machinery and preparing manufacturing instructions, like component tolerance.

Methodology

The finite element equations governing the lateral vibration of rotating systems, taking into account the element of Timoshenko's beam, consider eight degrees of freedom for each element, four degrees of freedom for transient lateral motion along the x and y axes and four degrees of freedom for rotational motion around these axes, is according to equation (1):

$$[M]\{\ddot{q}\} + [[C] + [G]]\{\dot{q}\} + [[K] + [H]]\{q\} = \{F\} \quad (1)$$

In this equation, [M], [C], [G], [K], and [H] are the matrices of mass, damping, gyroscope, stiffness, and circularity, respectively [7]. The vector q contains the rotating system degrees of freedom. Also, F is the external force acting on the rotating system, considered in this study as the unbalance forces.

Based on the Sobol decomposition method [8], the variance of the output function (indicating with Y), can be decomposed into Equation (2):

$$Var(Y) = \sum_{i=1}^d V_i + \sum_{i < j} V_{ij} + \dots + V_{12\dots d} \quad (2)$$

The contribution ratio of the principal variance of the X_i parameter to the output variance is denoted by S_i and is called the "first-order sensitivity indices" or the "main sensitivity indices" [1]. The terms of first-order

sensitivity and sensitivity of the interaction of variables are as equation (3):

$$S_i = \frac{V_i}{Var(Y)}, \quad S_{ij} = \frac{V_{ij}}{Var(Y)} \quad (3)$$

The "total sensitivity indices" are denoted by S_{Ti} and include all input effects X_i and is defined as equation (4),

$$S_{Ti} = \frac{E_{X_{\sim i}}(Var_{X_i}(Y|X_{\sim i}))}{Var(Y)} \\ = 1 - \frac{Var_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))}{Var(Y)} \quad (4)$$

In which $V_{\sim i} = Var_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))$ is variance contribution of all of the inputs except X_i in the output variance.

Discussion and Results

The studied compressor is a six-stage centrifugal compressor used in gas transmission lines to increase gas pressure. The compressor has an operating speed of 6,000 to 9,000 rpm and a critical speed of about 3,500 rpm. Manufacturing tolerance and assembly process are two primary sources of parameter uncertainty in the rotor, which lead to considerable change of rotor critical speed from one rotor to another, with the same nominal design parameters [5]. Because the temperature changes at the conducted tests were not significant, the properties of materials such as elastic modulus will not change much with respect to temperature and dimensional changes due to temperature changes. Therefore, in this study, the most important factor is the uncertainty due to the manufacturing tolerances. Eleven parameters, including the compressor material's geometric characteristics and properties, are considered candidates for the compressor's critical speed sensitivity, as shown in the table.1 .

Table 1. Selected parameters for sensitivity analysis

Parameter Number	Structural Definition	Considered Distribution
1	Impeller #1 mass	Uniform
2	Impeller #2 mass	Uniform
3	Impeller #3 mass	Uniform
4	Impeller #4 mass	Uniform
5	Impeller #5 mass	Uniform
6	Impeller #6 mass	Uniform
7	Pad clearance	Normal
8	Bearing clearance	Normal
9	Shaft elastic moduli	Uniform
10	Shaft mass density	Normal
11	Shaft diameter	Normal

The parameters with the highest uncertainty and the most decisive influence on the response have high sensitivity indices. Figure (1) shows the sensitivity indices of the candidate parameters obtained by the variance-based global sensitivity analysis.

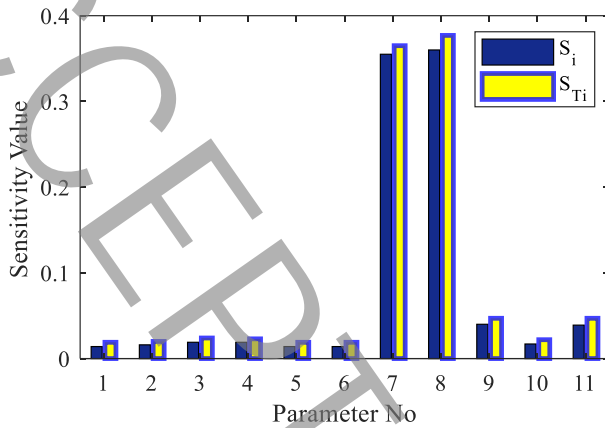


Figure 1. Sensitivity analysis result for uncertain parameters

According to the figure.1, the following results are obtained:

- The mass of the impellers has minor effects on the critical speed. Because the masses change is below one percent due to tolerances, the effect of impellers masses on the critical speed is negligible.
- The modulus of elasticity does not change much due to slight temperature variations and therefore has a low sensitivity coefficient and a low effect on dynamic behavior.
- The global sensitivity indices of the shaft diameter are also low due to the manufacturing tolerance.

However, using the local sensitivity method indicates high local sensitivity of the shaft diameter because it has a power of four in the moment of inertia. In contrast, the global sensitivity is low because the variation of the diameter or its tolerance is small due to manufacturing tolerance.

- According to the manufacturing tolerances, bearing clearance and pad clearance are two parameters with the highest sensitivity indices among others. This result was expected because the bearing and pad clearances significantly affect the dynamic coefficients of hydrodynamic bearings and the rotor critical speed [5, 7]. However, in addition to qualitative predictions, this analysis provides a quantitative criterion for identification of sensitive parameters.

Conclusion

Many researchers have considered uncertainty quantification in recent years. Increasing the dimensions and complexity of the structures leads to increasing

uncertainties in them and increases the need for random modeling of the systems. In this study, using the global sensitivity method, the uncertainty in the critical speed of an industrial rotating system is devoted to its input parameters. Variance-based global sensitivity is a powerful method to identify sensitive parameters affecting system output. Using this method to identify sensitive parameters on system output in the rotating system shows that one can identify effective parameters for model updating of the complex industrial rotors.

In this paper, a proper framework for sensitivity analysis of rotational systems, based on variance decomposition and Monte Carlo methods, is introduced. Uncertainty in the system output is quantitatively devoted to the parameters uncertainty and provides quantitative analysis and qualitative prediction for the designer.

The proposed method can be used to identify effective geometric parameters and determine manufacturing tolerances to improve the design of the rotating system.

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