

# Piezoelectric energy harvesting using a porous beam under fluid-induced vibrations

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## ABSTRACT

In this paper, the energy harvesting by porous beams exposed to the external fluid flow is studied. The electromechanical nonlinear differential equations of the transverse vibration behavior of porous beams exposed to external fluid flow are derived using the Euler-Bernoulli beam theory. A porous beam with concentrated mass which is equipped with a piezoelectric layer at its upper surface is considered energy harvesting. After numerically solving the governing nonlinear equations, the effect of different parameters on the generated energy is investigated. The results show that in the lock-in area, the maximum amount of energy is taken. Also, the porosity distribution has a significant effect on the maximum amplitude of the oscillations as well as the energy harvesting by the porous beam. In addition, for electrical resistance of 1000 k $\Omega$ , the maximum voltage generated for the beam with symmetrical porosity distribution in the form of wall stiffness, asymmetric porosity distribution, and uniform porosity distribution is equal to 0.39 V, 0.44 V, and 57 V, respectively, which indicates the highest energy harvesting capability of the beam with the porosity distribution of the third type.

## KEYWORDS

Energy harvesting, Porous beam, Fluid induced vibrations, Lock-in region.

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## Introduction

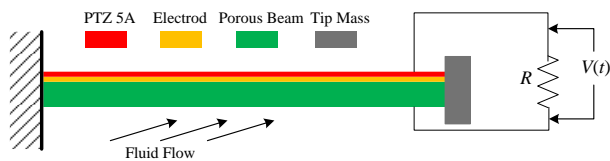
Defects in mechanical structures are one of the main reasons for the error in the mathematical modeling of mechanical systems. Porosity [1-3] is one of the most common defects in these structures. Avoiding porosity in parts made in the additive manufacturing process by metal 3D printers, which have also recently developed extensively, is inevitable [4-6]. Accordingly, considering these defects in the mathematical modeling of mechanical structures, especially beams, can increase the accuracy of modeling and achieve results consistent with experimental data.

Extensive research has been done on the production of electrical energy from the forced vibrations of the beam [7]. Li et al. [8] studied the energy harvesting capability of the bridge using the finite element method. Dai et al. [9] investigated energy harvesting by fluid-induced vibrations flow and the base excitation. Radgelchin et al. [10] investigated the electrical energy harvesting from a beam under basic excitation. Qi [11] analytically studied the energy harvesting from a functionally graded beam.

In this paper, using Euler-Bernoulli beam theory and considering the interaction of structure and fluid the coupling nonlinear differential equations of the motion governing the induced vibration behavior of porous beam fluid with piezoelectric layers as energy harvesting are extracted. The nonlinear equations are discretized using the Galerkin method, and finally, by numerically solving the discretized equations, the effect of different parameters on the vibration characteristics and energy harvesting of these beams is investigated.

## 2- Equations of motion

As shown in Fig. 1, the beam is a porous beam with a rectangular cross-section and a piezoelectric layer of PZT 5A is used on the upper surface of the beam as energy harvesting layers.



**Fig. 1** Configuration of piezoelectric energy harvesting with the core made of porous material

Three different porosity distributions are considered in terms of beam thickness. Continuous changes in Young's modulus ( $E$ ), shear modulus ( $G$ ), density ( $\rho$ ), and Poisson's ratio ( $\nu$ ) can be obtained using the following equations [12]:

$$E(z) = E_{\max} (1 - e_0 q(z)) \quad (1)$$

$$\nu(z) = 0.221 \tilde{p} + \nu_{\max} (0.342 \tilde{p}^2 - 1.21 \tilde{p} + 1) \quad (2)$$

$$\rho(z) = \rho_{\max} (1 - e_0 q(z)) \quad (3)$$

where  $q(z)$  represents the porosity distribution function in the direction of the beam thickness.  $e_0$  is the porosity coefficient of the beam is between zero and one.

Using dimensionless variables, the nonlinear equations governing the fluid-induced vibration behavior of the porous beam as an energy harvesting system in terms of dimensionless variables are obtained as follows:

$$-\alpha_4 \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} - \alpha_5 \frac{\partial}{\partial \hat{x}} \left( \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} \frac{\partial \hat{w}}{\partial \hat{x}} \right) - \alpha_1 \frac{\partial \hat{w}}{\partial \hat{x}} \left( \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} \frac{\partial \hat{w}}{\partial \hat{x}} \right) + \hat{\theta} \frac{\partial^2}{\partial \hat{x}^2} (V(t) [H(\hat{x}) - H(\hat{x} - 1)]) - (1 + \beta \delta (\hat{x} - 1)) \frac{\partial^2 \hat{w}}{\partial \tau^2} \quad (4)$$

$$+\alpha_6 \frac{\partial^4 \hat{w}}{\partial \tau^2 \partial \hat{x}^2} = c_L \hat{U}^2 \bar{q} - c_D \hat{U} \frac{\partial \hat{w}}{\partial \tau} \frac{\partial^2 \bar{q}(\hat{x}, \tau)}{\partial \tau^2} + \delta \Omega_{os} u [\bar{q}(\hat{x}, \tau)^2 - 1] \frac{\partial \bar{q}(\hat{x}, \tau)}{\partial \tau} + \Omega_{os}^2 u^2 \bar{q}(\hat{x}, \tau) = T \frac{\partial^2 \hat{w}(\hat{x}, \tau)}{\partial \tau^2} \quad (5)$$

$$V(\tau) - \alpha_9 \frac{dV(\tau)}{d\tau} = - \int_0^1 \left( \alpha_7 \frac{\partial^3 \hat{w}}{\partial \tau \partial \hat{x}^2} + \alpha_8 \frac{\partial^2 \hat{w}}{\partial \tau \partial \hat{x}} \frac{\partial \hat{w}}{\partial \hat{x}} \right) d\hat{x} \quad (6)$$

According to the Galerkin method, the hypothetical solution of the differential equation is considered as follows:

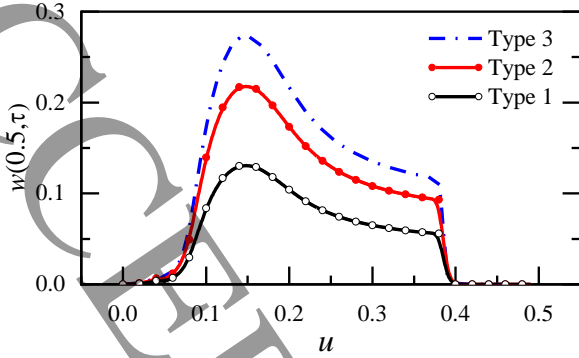
$$w(x, \tau) = \sum_{n=1}^N \varphi_n(x) y_n(\tau), \quad \bar{q}(x, \tau) = \sum_{n=1}^N \varphi_n(x) \chi_n(\tau) \quad (7)$$

By substituting Eq. (7) in Eqs. (5)-(6), the system of nonlinear differential equations with ordinary derivatives with unknown  $2N$  is obtained, which must be numerically solved in order to calculate the unknown parameters including beam transverse deflection, instantaneous oscillation coefficient, and generated voltage. By solving these equations using the Rangokota method, in the next section, the effect of different parameters is studied.

## 3- Results and discussion

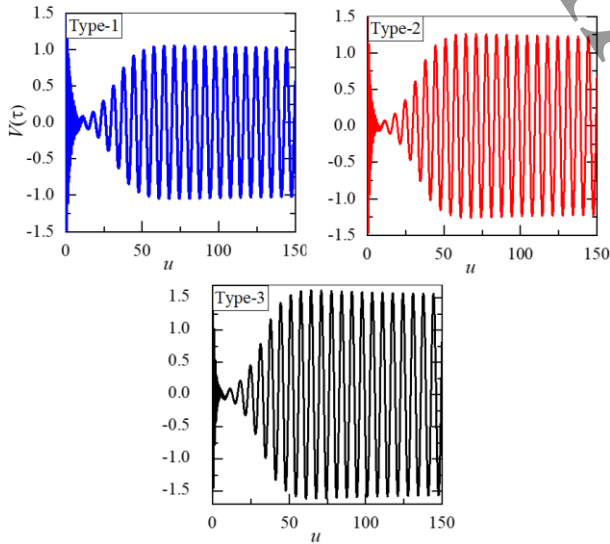
As can be seen from the results shown in Fig. 2, the amplitude of steady-state vibrations for the lock-in zone is greater than for the other regions. In addition, the results show that the porosity distribution has a significant effect on the lock-in area as well as the maximum amplitude of the porous beam oscillations. The maximum vibration amplitude for the third type distribution of porosity occurs in the velocity of  $u=0.015$ , the value of which is equal to 0.27. Also, the lock-in area for the beam with two other types of porosity distributions is created in this speed range, but the maximum amplitude created is different and for the first

and second type distributions is equal to 0.22 and 0.13, respectively.

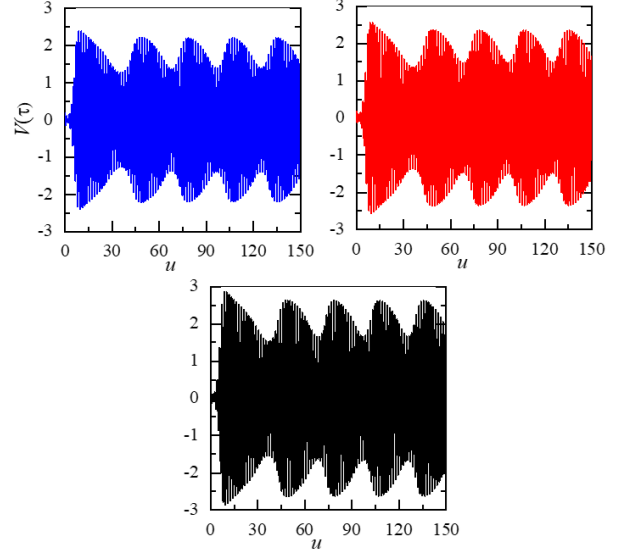


**Fig. 2** Maximum amplitude of oscillations of the midpoint of a porous beam in terms of external fluid flow velocity

Figs. 3 and 4 show the voltage generated by the porous beam for three different porosity distributions. At low fluid velocities, the output voltage, which is directly related to the system response, is oscillating with a constant amplitude. At the velocity of  $u=0.5$ , it causes different behavior in the time response of the porous beams. In this case, the beating phenomenon occurs in response. At this speed, the maximum voltage generated by the piezoelectric layers in the steady-state for the first type porosity distributions, second and third types is 2.38V, 2.52 V, and 2.87 V, respectively, which indicates the high energy production capability in the use of porosity beams with uniform porosity distribution (third type).



**Fig. 3** Time response of porous beam for different fluid flow velocities  $u=0.015$



**Fig. 4** Time response of porous beam for different fluid flow velocities  $u=0.5$

Table 1 provides a comparison between the maximum voltage obtained by the presented porous biomorphic beam in the present study and some of the models presented in the previous research. According to the results shown in this table, it can be seen that the use of porous beams has a very good ability in energy production and can replace similar existing systems.

**Table 1** Comparison between the maximum voltages of porous beams presented in the present study with some of the models presented in previous research

	Present work	Ref. [13]	Ref. [14]	Ref. [15]
Max. Voltage (V)	0.57	1.07	0.05	0.02

#### 4- Conclusion

In the present study, the effect of energy harvesting from porous beams exposed to external fluid flow was analyzed. A summary of the important results of the present study is:

- The porosity distribution has a significant effect on the time response as well as the maximum amplitude of the porous beams oscillations and affects the energy harvesting of this type of beam.

- The results show that the porosity distribution does not have a significant effect on the lock-in area, but the maximum amplitude of porous beam oscillations is strongly affected.

- The amount of energy that can be extracted in lock-in areas is much higher than in other areas, which is due to the existence of high dynamic strains in these areas.

- The results show that the maximum voltage of the porous beam is about 1.6 times higher than the

corresponding beam without porosity. Based on this, it can be concluded that the use of porous beams significantly increases the ability of energy harvesting due to fluid-induced vibrations, which is due to the high flexibility of porous beams.

## 5- References

- [1] R. Thomson, J. Hancock, Stress and strain fields near a contained porous imperfection in a plastically deforming matrix, *Res mechanica*, 16(2) (1985) 135-146.
- [2] K. Xie, Y. Wang, H. Niu, H. Chen, Large-amplitude nonlinear free vibrations of functionally graded plates with porous imperfection: A novel approach based on energy balance method, *Composite Structures*, 246 (2020) 345-367.
- [3] R. Kumhar, S. Kundu, M. Maity, S. Gupta, Analysis of interfacial imperfections and electro-mechanical properties on elastic waves in porous piezo-composite bars, *International Journal of Mechanical Sciences*, 187 (2020) 105-126.
- [4] R. Fu, S. Tang, J. Lu, Y. Cui, Z. Li, H. Zhang, T. Xu, Z. Chen, C. Liu, Hot-wire arc additive manufacturing of aluminum alloy with reduced porosity and high deposition rate, *Materials & Design*, 199 (2021) 34-51.
- [5] A. Sola, A. Nouri, Microstructural porosity in additive manufacturing: The formation and detection of pores in metal parts fabricated by powder bed fusion, *Journal of Advanced Manufacturing and Processing*, 1(3) (2019) 87-95.
- [6] D. Basu, Z. Wu, J.L. Meyer, E. Larson, R. Kuo, A. Rollett, Entrapped Gas and Process Parameter-Induced Porosity Formation in Additively Manufactured 17-4 PH Stainless Steel, *Journal of Materials Engineering and Performance*, 56 (2021) 1-8.
- [7] H. Farokhi, A. Gholipour, M.H. Ghayesh, Efficient Broadband Vibration Energy Harvesting Using Multiple Piezoelectric Bimorphs, *Journal of Applied Mechanics*, 87(4) (2020) 45-56.
- [8] A. Li, W. Zhao, S. Zhou, L. Wang, L. Zhang, Enhanced energy harvesting of cantilevered flexoelectric micro-beam by proof mass, *AIP Advances*, 9(11) (2019) 115305.
- [9] H. Dai, A. Abdelkefi, L. Wang, Piezoelectric energy harvesting from concurrent vortex-induced vibrations and base excitations, *Nonlinear Dynamics*, 77(3) (2014) 967-981.
- [10] M. Radgolchin, H. Moeenfard, Size-dependent piezoelectric energy-harvesting analysis of micro/nano bridges subjected to random ambient excitations, *Smart Materials and Structures*, 27(2) (2018) 12-24.
- [11] L. Qi, Energy harvesting properties of the functionally graded flexoelectric microbeam energy harvesters, *Energy*, 171 (2019) 721-730.
- [12] M.L. Facchinetti, E. De Langre, F. Biolley, Coupling of structure and wake oscillators in vortex-induced vibrations, *Journal of Fluids and Structures*, 19(2) (2004) 123-140.
- [13] F. Cottone, L. Gammaitoni, H. Vocca, M. Ferrari, V. Ferrari, Piezoelectric buckled beams for random vibration energy harvesting, *Smart materials and structures*, 21(3) (2012) 34-54.
- [14] A. Mehmood, A. Abdelkefi, M. Hajj, A. Nayfeh, I. Akhtar, A. Nuhait, Piezoelectric energy harvesting from vortex-induced vibrations of circular cylinder, *Journal of Sound and Vibration*, 332(19) (2013) 4656-4667.
- [15] A. Khatami, Response regime of nonlinear bistable energy harvester, *Moades Mechanical Engineering*, 18(5) (2018) 57-65.