Accurate calculation of nodal vectors in isogeometric analysis of shell structures, using Greville points

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ABSTRACT

In this paper, the isogeometric analysis of shell structures along with the optimal method for accurate calculation of nodal vectors is proposed. The Non-uniform rational B-spline is used for shell mid-surface description. According to the Reisner- Mindlin hypothesis, the director vectors at control points are needed for the interpolation of the rotations. The calculated nodal direct vectors must lead to exact interpolated director vectors on the shell surface. Hence, a method has been proposed in which the components of director vectors at control points are obtained by solving a system of equations on the whole patch. The system of equations is formed using known values of direction vectors at the Greville points. The accuracy of the proposed method has been investigated by using the results of the most common problem in shell analysis. Convergence behavior for displacement at the loading points has been studied in all solved problems for different order of basis functions and net of control points. The deformation results show better convergence behavior with increasing the regularity and order of basis functions. The Greville points are in a one-to-one correspondence with control points. Thus, the system of equations on these points leads to a unique solution for the nodal direction vectors, and the time to solve equations is significantly reduced.

KEYWORDS

Isogeometric analysis, Non-uniform rational B-spline, Reissner-Mindlin shells, Greville points, nodal vectors

1. Introduction

Generally, common engineering software uses two separate models for design and analysis. The design process is usually based on computer-aided design (CAD) techniques, such as the NURBS technique, while linear Lagrangian functions are usually used to analyze for finite element analysis. Therefore, changes in geometry or mesh require interaction between these two models, which leads to an increase in solution time. Using common geometry descriptions in industry can create a closer link between the two processes and reduce analysis time. The main idea of isogeometric analysis proposed by Hughes et al. [1, 2] is to use the basic functions of the geometric model to approximate the unknown function and analysis. The most common geometry descriptions are NURBS, which are often used in isogeometric analysis for better integration of design and analysis [3]. The aim of this paper is to provide a robust and efficient formulation for the analysis of shell structures using isogeometric analysis such as the NURBS technique.

According to the Mindlin-Reisner shell formulation, the local coordinate system must be defined at any point on the physical surface. Since in the NURBS- based formulation the control points are not located on the physical surface, an adapted techniques must be used to define a director vector at the control points. Therefore, in conventional methods, each control point is mapped into a geometric point on the surface and the vectors are defined at this point [4,5]. Kang and Youn calculated the exact surface normal vectors and their analytic derivatives in gauss points [6]. Numerous maps have been presented by Adams et al. [7]. This mapping leads to an error that only is covered by using a sufficiently fine mesh. Recently, the isogeometric shell analysis with exactly calculated direction vectors was researched by Dornisch et al [8].

In this paper, like to the proposed method by Dornich et al, the direction vectors are obtained by solving the system of equations on patch with a difference that the director vectors at the Greville points are considered as known values of the system of equation. Due to the one-to-one correspondence between the control points and the Greville points, this method leads to a unique solution. The order of NURBS and the number of control points are increased using the k- refinement technique and the convergence behavior has been investigated for common shell problems in literature.

2. Isogeometric shell formulation

The employed shell formulation is based on the degenerated shell approach. The degeneration process is based on Reissner-Mindlin assumption and reducing the 3D field approach to a 2D one in mid-surface. The director vectors should be defined at points on the geometric surface but in spline-based approach, control points may not be located on the geometric surface. Therefore, the control points cannot directly be used to define the local coordinate system and consequently the rotational degrees of freedom. Hence, for each control point it is necessary to somehow find a corresponding point on the mid-surface of the shell and mount the aforementioned orthogonal vectors on it. There are different ways to find this point, a simple way is to map each control point into the nearest point on the geometric surface.

Here the direction vectors are obtained by solving the system of equations on patch with a difference that the director vectors at the Greville points, are considered as known values of the equations system. The system of equation is defined as:

$$A_{ij}^{Gr} = \sum_{I=1}^{n_{Gr}} R_I A_{ijI}^{cp} \quad i,j = 1,2,3 \tag{2}$$

In which A_{ij} denotes the component *j* of basis vector *i*.

3. Results and Discussion

The pinched hemisphere shell with an 18◦ circular cutout is analyzed. The spherical shell is subjected to four radial point loads P, at 90◦ intervals.

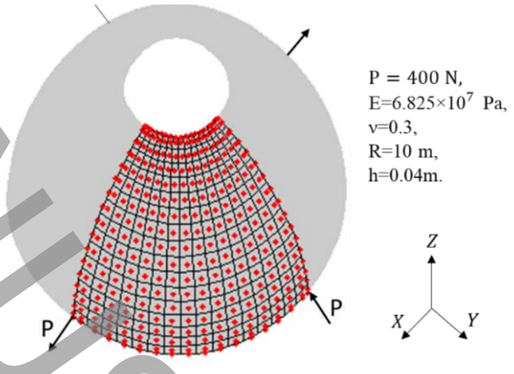

Fig 1. A quarter model of hemispherical shell subjected to radial forces

Due to symmetry conditions the one quarter of the shell is modeled, as shown in Figure 1. The convergence behavior of displacement at loading point for all orders of NURBS is presented in Figure 2. It can be seen that presented method leads to improved convergence behavior. Such that for coarse mesh, rising the orders of NURBS leads to better accuracy whereas In a way the director vectors are calculated at so- named anchor points that consider as collocated method in Figure 2.

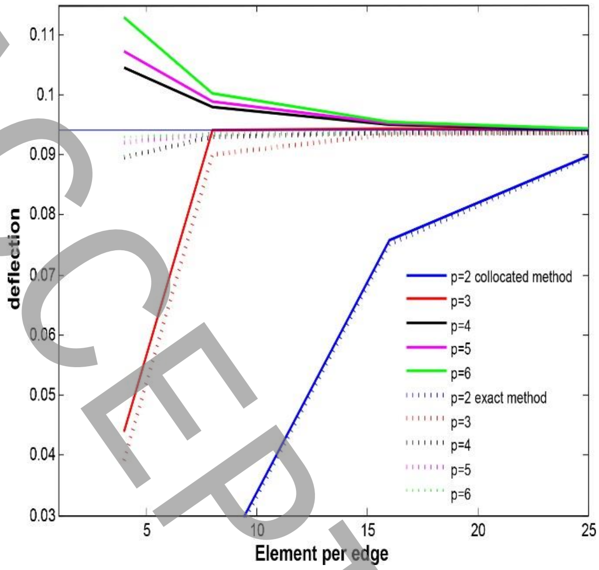

Fig 2. Comparison of deformation convergence behavior of hemispherical shell for various order of the NURBS

The deflection of loading point and time to compute of director vectors for two different methods are compared in Table 1. It should be noted for presented data in Table 1, that the definition of equations at Guass points is considered as method 1 and definition of equations at Greville points is considered as method 2. The comparison of time to compute for two methods show that the Greville points are more suitable for definition of system of equations. Whereas the time to compute is decreased, the accuracy of results is preserved.

Table 1. **Comparison of deflections and time to calculate vectors for two mentioned methods**

Pinched hemispherical shell				
P	Method 1		Method2	
	Deflection	Time(s)	Deflection	Time(s)
3	0/0925	3/87	0/0925	0/4
4	0/0934	5/41	0/0934	0/4
5	0/0936	7/59	0/0936	0/5

4. **Conclusions**

The shell formulation based on Reisner-Mindlin theory is expressed in IGA framework for the analysis of shell structures. The NURBS and K-method are used for exact geometry description and discretization of shell mid-surface. In shell formulation the local coordinate system must be defined at any point on the physical surface. Since in the NURBS- based formulation the control points are not located on the physical surface, an adapted techniques must be used to define a director vector at the control points. Therefore, in conventional methods, each control point is mapped into a geometric point on the surface and the vectors are defined at this point. This mapping leads to an error that only is covered by using a sufficiently fine mesh. Here, the director vector are obtained by solving the system of equations defined on the Greville points. These points are one-to-one correspondence with control points that lead to a unique solution and reduces the solution time. On the other hand, the interpolation error don't increase for coarse mesh and high orders of NURBS. The study of convergence behavior for critical shell problems confirm this conclusion In order to validate the results and the accuracy of this method, the displacements for the hemisphere and cylinder problems were compared with other references that are in high agreement. By comparing the computation time of vectors, it can be concluded that the Greville points for defining of the system of equations are a good choice especially for problems with large computational efforts such as nonlinear solutions.

5. **References**

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