Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 53(Special Issue 1) (2021)137-140
DOI: 10.22060/mej.2019.16538.6383

Nonlinear Transient Heat Transfer Analysis Using Two Integration Methods with Different Distributions of Integration Points in the Domain in a Meshless Formulation

S. Kooshki[1], M. Khodadad[1*] , A. Khosravifard[2]

[1] Department of Mechanical Engineering, Yazd University, Yazd, Iran
[2] Department of Mechanical Engineering, Shiraz University, Shiraz, Iran

ABSTRACT: In this article, the transient heat transfer problem with both convection and radiation boundary conditions is studied. The meshless radial point interpolation method is implemented in this numerical study. Also, two integration methods, the Cartesian transformation method and the Gaussian quadrature method which uses background cells, are employed for computation of the domain integral. First, a homogenous medium with both convection and radiation boundary conditions is considered. The temperature distribution obtained by the proposed meshless method is compared with the analytical solution for a heat transfer problem and excellent agreement is observed. Then, a number of example problems in a layered composite and a functionally graded sample with both convection and radiation boundary conditions are solved and the temperature results are compared with those of ABAQUS software. Through the numerical examples it is observed that using the cartesian transformation method in comparison with the background cell method in convection boundary conditions reduces the error to half and in radiation boundary conditions reduces the error to one-quarter. This numerical method is a meshless method which does not require any background mesh. Moreover, the amount of error using the background cell method in problems with radiation boundary conditions is more than those with convection boundary conditions. This shows the advantage of using the cartesian transformation method in problems with radiation boundary condition which have a higher degree of nonlinearity, due to the temperature-dependent boundary conditions.

Review History:

Received: 2019-06-11
Revised: 2019-09-25
Accepted: 2019-11-05
Available Online: 2019-12-09

Keywords:

Convection-radiation boundary conditions
Meshless method
Nonlinear transient heat transfer
Cartesian transformation method
Background cell

1. INTRODUCTION

Nowadays, new materials like layered composites and Functionally Graded Materials (FGMs) have many applications because of their commercial and environmental advantages [1]. Because of a gradual change of microstructure in FGMs, their macroscopic properties such as thermal conductivity and specific heat, change gradually. Therefore, FGMs are very good candidates for high-temperature applications like Thermal Barrier Coatings (TBCs), combustion chambers, etc. [2, 3]. Consequently, the study of numerical methods which can evaluate their thermal behavior is valuable.

Since FGMs are non-homogenous, many analytical solutions are not applicable to them [4]. Solving the transient heat transfer problem in FGMs needs a powerful numerical method. Also, in temperatures higher than 600°C, radiation should be considered. However, transient heat conduction with radiation has not been the subject of many studies [5]. Therefore, the study of transient heat conduction problems with radiation Boundary Conditions (BCs) is valuable.

In this study, the meshless Radial Point Interpolation Method (RPIM) is adopted for numerical analyses. This method was first introduced by Wang and Liu [6, 7]. The RPIM is based on the local weak formulation; consequently, it should be combined with an integration method for computation of the domain integrals. In this study, the application of the traditional method of the Gaussian Quadrature (GQ) by the use of background cells is compared with the application of the Cartesian Transformation Method (CTM) [8] in RPIM formulation. Using the CTM for calculation of the domain integrals results in a truly-meshless Radial Point Interpolation Method (t-RPIM) [4, 9] which does not require any background mesh.

Firstly, the transient temperature responses in a homogenous domain with both convection and radiation BCs obtained by the t-RPIM are validated by analytical temperature responses of the same problem. Then, a layered composite and a Functionally Graded (FG) sample made of Al_2O_3 and ZrO_2 powders are considered. The transient temperature responses in these samples with both convection and radiation BCs using both the GQ method and the CTM in RPIM formulation are calculated and the results are compared to those obtained from ABAQUS software, and an excellent agreement is observed.

2. METHODOLOGY

The transient heat transfer problem in a non-homogenous and isotropic medium with temperature-dependent thermal properties is considered. The governing differential equation without internal heat sources in the domain can be written as:

*Corresponding author's email: Khodadad@yazd.ac.ir

Copyrights for this article are retained by the author(s) with publishing rights granted to Amirkabir University Press. The content of this article is subject to the terms and conditions of the Creative Commons Attribution 4.0 International (CC-BY-NC 4.0) License. For more information, please visit https://www.creativecommons.org/licenses/by-nc/4.0/legalcode.

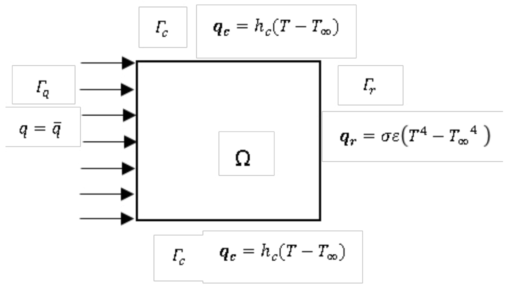

Fig. 1. Different boundary conditions in the domain problem.

$$\nabla.(k\nabla \boldsymbol{T}) = \rho \times c \times \partial \boldsymbol{T} / \partial t \tag{1}$$

in which $\rho(\mathbf{x})$ is the density, $k(\mathbf{x},T)$ is the thermal conductivity and $c(\mathbf{x},T)$ is the specific heat. The corresponding transient heat transfer problem with convection and radiation BCs (Fig. 1) is considered. In Fig. 1, h_c is the convection heat transfer coefficient, T_∞ is the ambient temperature, σ is Stefan-Boltzmann constant and ε is the absorptivity coefficient of the surface, which is between 0 and 1.

By calculation of the RPIM shape functions for the transient heat transfer problem, and substituting them in the weak form of Eq. (1), the following system of equations is obtained in which $\{\boldsymbol{T}\}$ is the temperature vector and $\{\boldsymbol{F}(t,T)\}$ is the thermal load vector, both with the dimension of $N\times1$. $[\boldsymbol{M}]$ is mass matrix and $[\boldsymbol{K}(T)]$ is the thermal stiffness matrix, both with the dimension of $N\times N$, where N is the total number of meshless nodes in the domain [8].

$$[\boldsymbol{M}]\{\dot{\boldsymbol{T}}\} + [\boldsymbol{K}(T)]\{\boldsymbol{T}\} = \{\boldsymbol{F}(t,T)\} \tag{2}$$

Eq. (2) is solved in the time domain using the Crank-Nicolson scheme [10]. Mass, stiffness, and load matrices are in the form of the domain and boundary integrals as follows, where φ_i is the shape function of node i.

$$M_{ij} = \int_\Omega \rho(x,T)c(x,T)\varphi_i\varphi_j \,\mathrm{d}\Omega \tag{3}$$

$$K_{ij} = \int_\Omega K(x,T)\left[\frac{\partial\varphi_i}{\partial x}\frac{\partial\varphi_j}{\partial x} + \frac{\partial\varphi_i}{\partial y}\frac{\partial\varphi_j}{\partial y}\right]d\Omega + \int_{\Gamma_c} h_c\varphi_i\varphi_j d\Gamma_c + \int_{\Gamma r} h_r\varphi_i\varphi_j d\Gamma_r \tag{4}$$

$$F_i = -\int_{\Gamma_q} \bar{q}\varphi i d\Gamma_q + \int_{\Gamma_c} h_c T_\infty \varphi_i d\Gamma_c + \int_{\Gamma_r} h_r T_\infty \varphi_i d\Gamma_r \tag{5}$$

$$h_r = \sigma\varepsilon[(T + T_\infty)(T^2 + T_\infty^2)] \tag{6}$$

h_r in Eqs. (4) to (6) is the effective radiation heat transfer coefficient [11] which is a non-linear function of temperature. In the t-RPIM, the domain integrals in Eqs. (3) and (4) are calculated using the CTM. In traditional RPIM, these integrals are calculated using the GQ. In this work, the analyses are performed using both of these integration methods in the RPIM and the obtained results are compared.

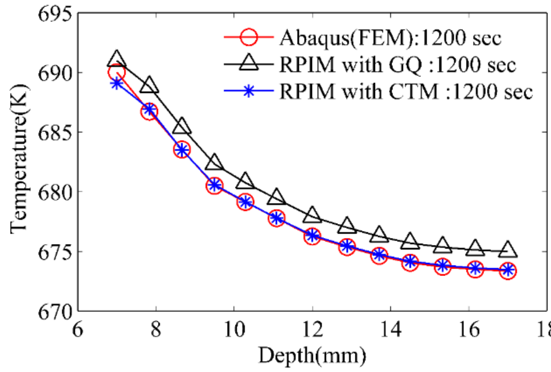

Fig. 2. Comparison of the temperature-depth profile in the layered sample with convection boundary condition.

3. RESULTS AND DISCUSSION

A transient heat transfer problem in a homogenous domain with both convection and radiation BCs is considered. The analytical temperature response in this problem calculated using the Fourier expansion [12] has an excellent agreement with the calculated t-RPIM temperature responses which verifies the validity of the t-RPIM.

Also, a similar problem is studied in a layered composite and an FG sample. The obtained temperature responses are compared with those obtained from ABAQUS software and good agreement between them is observed. Moreover, these problems are solved using the RPIM combined with the GQ method for domain integrals calculation and the calculated temperature responses are compared with those obtained using the t-RPIM and ABAQUS. It is observed that the calculated temperatures using the t-RPIM have better agreement with those calculated using the GQ which shows the better ability of the t-RPIM in dealing with transient heat transfer problems. A comparison of these temperature-depth profiles for the layered sample along a horizontal path with all the boundaries subjected to convection heat transfer is shown in Fig. 2, as an example.

4. CONCLUSION

The t-RPIM is capable of solving the transient heat transfer problem both in homogenous and non-homogenous media with convection and radiation BCs. In radiation BC, the problem has a higher degree of nonlinearity, due to temperature-dependent BCs. However, the t-RPIM can calculate temperature responses with less than 2% difference in comparison with analytical (homogenous) and ABAQUS results.

Moreover, a comparison of the temperature responses obtained using the CTM and the GQ method with those obtained by ABAQUS shows that by using the CTM in the meshless solution, the error reduces to half and one quarter in convection and radiation BCs, respectively. It should be mentioned that the number of integration points used in the CTM is less than those in the GQ method. Therefore, this improvement in the accuracy of the calculated temperature responses is obtained without using more integration points. This is a positive point for the CTM since it requires a smaller number of integration points and therefore reduces the computational time.

REFERENCES

[1] R. Joshi, S. Alwarappan, M. Yoshimura, V. Sahajwalla, Y. Nishina, Graphene oxide: the new membrane material, Applied Materials Today, 1(1) (2015) 1-12.

[2] Y. Torres, P. Trueba, J. Pavón, E. Chicardi, P. Kamm, F. García-Moreno, J. Rodríguez-Ortiz, Design, processing and characterization of titanium with radial graded porosity for bone implants, Materials & Design, 110 (2016) 179-187.

[3] S. Naga, M. Awaad, H. El-Maghraby, A. Hassan, M. Elhoriny, A. Killinger, R. Gadow, Effect of La2Zr2O7 coat on the hot corrosion of multi-layer thermal barrier coatings, Materials & Design, 102 (2016) 1-7.

[4] A. Khosravifard, M. Hematiyan, L. Marin, Nonlinear transient heat conduction analysis of functionally graded materials in the presence of heat sources using an improved meshless radial point interpolation method, Applied Mathematical Modelling, 35(9) (2011) 4157-4174.

[5] W. Ge, C. Zhao, B. Wang, Thermal radiation and conduction in functionally graded thermal barrier coatings. Part I: Experimental study on radiative properties, International Journal of Heat and Mass Transfer, 134 (2019) 101-113.

[6] J. Wang, G. Liu, On the optimal shape parameters of radial basis functions used for 2-D meshless methods, Computer methods in applied mechanics and engineering, 191(23) (2002) 2611-2630.

[7] J. Wang, G. Liu, A point interpolation meshless method based on radial basis functions, International Journal for Numerical Methods in Engineering, 54(11) (2002) 1623-1648.

[8] A. Khosravifard, M.R. Hematiyan, A new method for meshless integration in 2D and 3D Galerkin meshfree methods, Engineering Analysis with Boundary Elements, 34(1) (2010) 30-40.

[9] T. Bui, A. Khosravifard, C. Zhang, M. Hematiyan, M. Golub, Dynamic analysis of sandwich beams with functionally graded core using a truly meshfree radial point interpolation method, Engineering structures, 47 (2013) 90-104.

[10] J.N. Reddy, An Introduction to the Finite Element Method, (1993) 227–230.

[11] J.N. Reddy, D.K. Gartling, The finite element method in heat transfer and fluid dynamics, CRC press, 2010.

[12] Y. Cengel, Heat and mass transfer: fundamentals and applications, McGraw-Hill Higher Education, 2014.

HOW TO CITE THIS ARTICLE

S. Kooshki, M. Khodadad, A. Khosravifard, Nonlinear Transient Heat Transfer Analysis Using Two Integration Methods with Different Distributions of Integration Points in the Domain in a Meshless Formulation, Amirkabir J. Mech Eng., 53(Special Issue 1) (2021) 137-140.

DOI: 10.22060/mej.2019.16538.6383