Propagation of thermomechanical waves in annular disks made of functionally graded materials under thermal shock

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ABSTRACT

In this paper, using the coupled Lord-Shulman generalized thermoelasticity theory and considering the nonlinear thermal effects, the thermoelastic behavior of annular disks made of functionally graded materials (FGMs) under internal thermal shock is investigated. To this end, the governing equations of problem are first derived within the framework of polar coordinates system. It should be noted that the energy equation is kept in its original nonlinear form in this derivation process. The solution procedure is then presented based on the generalized differential quadrature method. In the numerical results, the effects of important parameters such as FG index and magnitude of applied thermal shock on the propagation of thermomechanical waves in the disks are studied. The results show that with increasing the FG index, displacement and stress decrease as time evolves. Also, with presenting results for various magnitudes of thermal shock it is shown that conducting a nonlinear thermal analysis is necessary when the thermal shock magnitude is considerable. In addition, it is revealed that the fluctuations in the temperature are reduced as the relaxation time decreases. Moreover, increasing this parameter leads to the temperature variations, whereas the frequency of system decreases.

KEYWORDS

Functionally graded material, Generalized differential quadrature method, Annular disk, Generalized thermoelasticity, Thermal shock.

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1. Introduction

Engineering structures are under thermal shock in various applications for which the analysis of thermomechanical phenomena is of great importance. Using the Fourier heat conduction law may lead to incorrect results when a solid is subjected to thermal shock. This is because temperature and temperature gradient are high, while the time period of operation is on the order of picosecond. Hence, the Fourier law fails to correctly capture heat wave propagation phenomena, and the speed of thermal wave propagation becomes infinite that is physically incorrect. Accordingly, some generalized thermoelasticity theories have been proposed. The thermoelasticity developed by Lord and Shulman [1] is the first and simplest theory in this area which is known as the L – S theory. According to this theory, the wavy motion of temperature is captured through inserting a simple relaxation time into the conventional Fourier law. A literature survey shows that there are numerous works on the thermoelastic analysis of various structures using the L-S theory (e.g. [2-4]). In the present paper, an efficient numerical approach is proposed for the nonlinear generalized thermoelasticity problem of annular disks made of FGMs under thermal shock. The governing equations in polar coordinates are first derived based on the L-S theory considering nonlinear thermal effects. A numerical solution approach is then developed based on the GDQ technique. Effects of important parameters including FG gradient index and thermal shock on the coupled thermomechanical response of disks are investigated.

2. Methodology

In the absence of body forces and considering axisymmetric displacements, the radial equation of motion is written as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \ddot{u} \tag{1}$$

The governing equations of motion are expressed as

$$\begin{aligned}&\left(\bar{\lambda} + 2\mu\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + \\ &\frac{\partial\left(\bar{\lambda} + 2\mu\right)}{\partial r}\frac{\partial u}{\partial r} + \frac{u}{r}\frac{\partial \bar{\lambda}}{\partial r} - \rho\frac{\partial^2 u}{\partial t^2} \\ &-\left(\bar{\beta}\frac{\partial}{\partial r} + \frac{\partial \bar{\beta}}{\partial r}\right)\left(T - T_0\right) = 0\end{aligned} \tag{2}$$

For the thermally nonlinear first law of thermodynamics in the disk according to the L–S theory one can write

$$\begin{aligned}&K\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{\partial T}{\partial r}\frac{\partial K}{\partial r} - \\ &\bar{\beta}T\left(\frac{\partial \dot{u}}{\partial r} + \frac{\dot{u}}{r}\right) - t_0\bar{\beta}\dot{T}\left(\frac{\partial \dot{u}}{\partial r} + \frac{\dot{u}}{r}\right) - \\ &t_0\bar{\beta}T\left(\frac{\partial \ddot{u}}{\partial r} + \frac{\ddot{u}}{r}\right) - c\rho\dot{T} - t_0 c\rho\ddot{T} = 0\end{aligned} \tag{3}$$

The dimensionless form of thermomechanical governing equations is then expressed as

$$\begin{aligned}&\frac{\bar{\lambda} + 2\mu}{\bar{\lambda}_m + 2\mu_m}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) + \\ &\frac{1}{\bar{\lambda}_m + 2\mu_m}\frac{\partial\left(\bar{\lambda} + 2\mu\right)}{\partial r}\frac{\partial u}{\partial r} + \frac{1}{\bar{\lambda}_m + 2\mu_m}\frac{u}{r}\frac{\partial \bar{\lambda}}{\partial r} - \\ &\frac{\rho}{\rho_m}\frac{\partial^2 u}{\partial t^2} - \frac{1}{\bar{\beta}_m}\left(\bar{\beta}\frac{\partial}{\partial r} + \frac{\partial \bar{\beta}}{\partial r}\right)\theta = 0 \\ &\frac{k}{k_m}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) + \frac{1}{k_m}\frac{\partial \theta}{\partial r}\frac{\partial k}{\partial r} - \frac{\bar{\beta}}{\bar{\beta}_m}\hat{C}\left(\theta + 1\right) \\ &\left(\frac{\partial^2 u}{\partial t\,\partial r} + \frac{\partial u}{r\,\partial t} + t_0\frac{\partial^3 u}{\partial t^2\partial r} + t_0\frac{\partial^2 u}{r\,\partial t^2}\right) \\ &-\frac{\bar{\beta}}{\bar{\beta}_m}\hat{C}t_0\frac{\partial \theta}{\partial t}\left(\frac{\partial^2 u}{\partial t\,\partial r} + \frac{\partial u}{r\,\partial t}\right) - \\ &\frac{\rho c}{\rho_m c_m}\frac{\partial \theta}{\partial t} - \frac{\rho c}{\rho_m c_m}t_0\frac{\partial^2 \theta}{\partial t^2} = 0\end{aligned} \tag{4}$$

Using the GDQ technique, the discretized equations can be represented in the following matrix form

$$\begin{aligned}&\begin{bmatrix} M^{uu} & M^{u\theta} \\ M^{\theta u} & M^{\theta\theta}\end{bmatrix}\begin{Bmatrix}\ddot{u} \\ \ddot{\theta}\end{Bmatrix} + \begin{bmatrix} C^{uu} & C^{u\theta} \\ C^{\theta u} & C^{\theta\theta}\end{bmatrix}\begin{Bmatrix}\dot{u} \\ \dot{\theta}\end{Bmatrix} + \\ &\begin{bmatrix} K^{uu} & K^{u\theta} \\ K^{\theta u} & K^{\theta\theta}\end{bmatrix}\begin{Bmatrix}u \\ \theta\end{Bmatrix} = \begin{Bmatrix}F^{u} \\ F^{\theta}\end{Bmatrix}\end{aligned} \tag{5}$$

Boundary conditions are also given by

$$\begin{aligned}&r = a : u = 0, \quad -\frac{\partial T}{\partial r} = q_{in} \\ &r = b : \sigma_{rr} = 0, \quad T = T_0\end{aligned} \tag{6}$$

The boundary conditions are directly imposed on Eq. (5), and then, equations of motion and energy are written as

$$M\left(X\right)\ddot{X}+C\left(X,\dot{X}\right)\dot{X}+KX=F \tag{7}$$

The Newmark direct integration scheme and the Picard iterative technique are finally used to find the solution of problem.

3. Results and Discussion

Fig. 1 indicates the temporal evolution of radial displacement, temperature, hoop and radial stresses at the middle of a disk subjected to thermal shock. The dimensionless thermal shock, non-dimensional relaxation time and the coupling parameter are taken as 3, 0.64 and 0.0082594, respectively. Moreover, for the validation purpose with the results given in [4], the FG index is taken as zero. It is observed that the results are in good agreement with those reported in [4]. Also, this figure shows the wavy motion of temperature history which is owing to using the L-S theory.

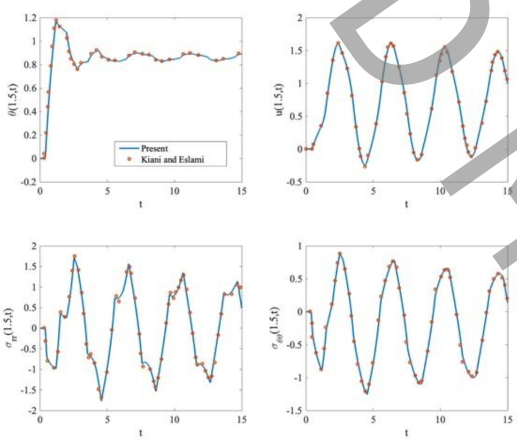

Figure 1. Temporal evolution of radial displacement, temperature, hoop and radial stresses at the middle of a homogenous disk

The influence of FG index on the displacement history, temperature and stresses waves of disk are highlighted in Fig. 2. Three values are considered for the FG index including 0.0, 0.2 and 0.5. It is seen that the amplitude of displacement and stresses decreases by the evolution of time as the FG index gets larger.

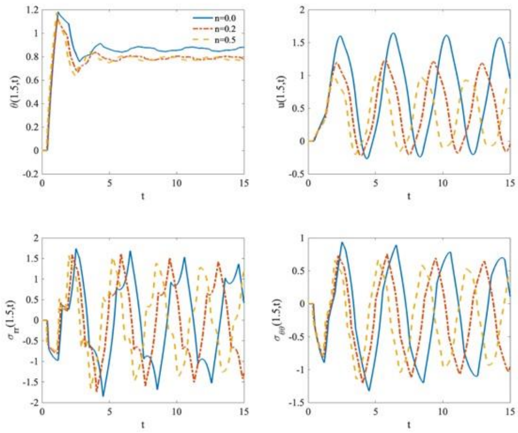

Figure 2. Temporal evolution of radial displacement, temperature, hoop and radial stresses at the middle of an FG disk for various FG indexes

4. Conclusions

Main conclusions of the paper can be summarized as:

The amplitude of displacement and stresses decreases considerably and frequency of oscillations increases when the FG index gets larger.

With the decrease of relaxation time, the fluctuations of temperature decreases.

Increase of thermal shock magnitude leads to the increase of displacement amplitude and stresses. However, it does not affect the frequency of oscillations.

5. References

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