Dynamic Analysis of Micro -Scale Parallelogram Flexures Using Beam Constraint Model and Modified Strain Gradient Theory

Mohammad Arhami[1], Hamid Moeenfard[1,2*]

[1]Department of Mechanical Engineering, Ferdowsi University of Mashhad
[2]Center of excellence in soft computing and intelligent information processing, Ferdowsi University of Mashhad

ACCEPTED MANUSCRIPT

ABSTRACT

In this paper, the dynamic behavior of a small-scale parallelogram (P) flexure is studied. First, using the beam constrain model and the modified strain gradient theory, the nonlinear strain energy of a small-scale beam is obtained in terms of its tip displacements. This energy expression is utilized to derive the strain energy of a P-flexure. Then the governing dynamic equations of motion are derived using Lagrange equations and is linearized around the operating equilibrium point. This linear model is employed to determine the allowable forces which do not lead to instability of the system. Moreover, the natural frequencies of the system is also extracted and the size effect as well as the static components of the applied loads on them is studied in detail. It is observed that with reducing the dimensions, the normalized transverse natural frequency of the system is increased. However, since there is no strain gradient in an axial mode, the axial normalized frequency is remailed constant with reducing the dimensions of the system. Moreover, it was observed that the tensile static forces lead to an increase, and transverse forces lead to a decrease in normalized natural frequency of the system. The procedure utilized for dynamic modeling of parallelogram flexures in this paper can be further extended for modeling more complex flexure systems.

KEYWORDS

Parallelogram flexure, Beam constraint model, Dynamic analysis, Modified strain gradient theory, stability analysis

---

[*] Corresponding Author's Email: h_moeenfard@um.ac.ir

ACCEPTED MANUSCRIPT

1. Introduction

Flexure mechanisms are kind of mechanisms that instead of using classical joints, employs the elastic deformation of its elements for providing the desired motion. Since fabricating classical mechanisms in small dimensions is very difficult, the main advantage of the flexure mechanisms is their application in small scales. Miniaturization provides the possibility of reducing the dimensions of the currently available devices. Moreover, it offers the possibility of fabricating new systems with unique applications. Among various examples of small-scale systems, one can point to the position sensors [1], accelerometers [2], force sensors [3], resonators [4], pressure gauges [5] gyroscopes [6] and micromirrors [7]. Most of the micro-scale mechanisms are fabricated using small-scale parallelogram (P) flexures which in turn are made up of two slenderness parallel beams connected to a moving stage. This element has very large stiffness in the constraint (rotational and axial) directions while presents very low stiffness in the transverse direction. This specification has made them very suitable for applications in positioning systems. So, dynamic analysis of these elements is of primary importance.

Beam constraint model (BCM) is a simple yet efficient approach for analysis of flexure mechanisms. The base of this method was first presented by Awtar and Slocum [8]. Then Awtar and Sen [9, 10] extended this method by presenting a closed-form expression for the nonlinear strain energy of a beam in terms of its tip displacements. This innovation made this method very suitable for analysis of more complex flexure units. Based on BCM, the static [11-13] and vibration [14-16] behavior of many flexure systems were studied. However, all these researches were based on classical elasticity theory which is not sufficiently accurate in small scale systems. So, the objective of the current research is to extend the BCM to micro dimensions using modified strain gradient method and then using it for dynamic analysis of micro-scale P-flexures.

2. Methodology

Figure 1, shows the schematic view of a P-flexure with a rigid motion stage, under the effect of end loads F_X, F_Z and M_Y. The length, width and thickness of the beams are respectively L, b and h.

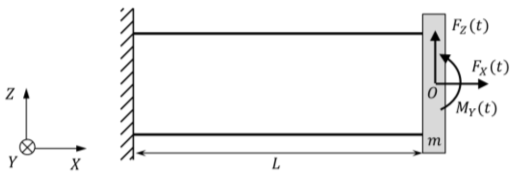

Figure 1: A micro-scale P-flexure

In P-flexures, the rotation of the stage is very small and can be easily neglected [16]. In this condition, the transverse and axial displacement components of the beams will be identical to those of the point O. Moreover, it can be shown that the normalized nonlinear strain energy of a P-flexure using BCM and modified strain gradient theory can be obtained as

$$v_P = a_1 \frac{\left(u\left(\hat{t}\right) + \frac{1}{2}k_{11}^{(1)} w^2\left(\hat{t}\right)\right)^2}{1 - a_1 k_{11}^{(2)} w^2\left(\hat{t}\right)} + k_{11}^{(0)} w^2\left(\hat{t}\right) \qquad (1)$$

where u and w are respectively the normalized axial and transverse displacements of O and $\hat{t}$ is the normalized time. Additionally, a_1, $k_{11}^{(0)}, k_{11}^{(1)}$ and $k_{11}^{(2)}$ are some constants that depend on the geometry of the system. Using dimensional form of (1) as the potential and $T_P = mL^2\left(\left(du/dt\right)^2 + \left(dw/dt\right)^2\right)/2$ as the kinetic energy of the system, Lagrange equations can be employed to derive the normalized equations of motion as

$$\ddot{u} + \frac{2u + k_{11}^{(1)} w^2}{1/a_1 - k_{11}^{(2)} w^2} = f_x \qquad (2)$$

$$\ddot{w} + \left(2k_{11}^{(0)} + k_{11}^{(1)}\left(\frac{2u + k_{11}^{(1)} w^2}{1/a_1 - k_{11}^{(2)} w^2}\right) + k_{11}^{(2)}\left(\frac{2u + k_{11}^{(1)} w^2}{1/a_1 - k_{11}^{(2)} w^2}\right)^2\right) w = f_z \qquad (3)$$

where f_x and f_z are respectively the normalized form of F_X and F_Z shown in Figure 1. Equations (2) and (3) can be linearized around the operating point as

$$[\mathcal{M}]\begin{Bmatrix}\ddot{\hat{u}} \\ \ddot{\hat{w}}\end{Bmatrix} + [\mathcal{K}]\begin{Bmatrix}\hat{u} \\ \hat{w}\end{Bmatrix} = \begin{Bmatrix}\hat{f}_x \\ \hat{f}_z\end{Bmatrix} \qquad (4)$$

where the mass matrix $[\mathcal{M}]$ is the identity, and $[\mathcal{K}]$ is the symmetric stiffness matrix. By observing the location of the poles of the transfer function of the system, its stability limit can be determined.

3. Results and Discussion

In Figure (2), the stability limit along with the map of $w < 0.15$ within which the BCM is valid, is depicted for a P-flexure for the case of $l = 17.6\ \mu\text{m}$. Also, the natural frequencies of the system and their dependence to h is depicted in Figure 3. It is observed that as h is increased, the results of the proposed model tend to those of classical BCM. However, at small dimensions, there is a remarkable deviation between the models.

3- Conclusion

The objective of the current research is analysis of dynamic behavior of P-flexures as the flexure module utilized in most compliant mechanisms. To this end, first the nonlinear strain energy of the P-flexure is determined using the BCM and modified strain gradient theory. Then, Lagrange equations were employed to determine the governing equations of motion. The linearized form of these equations was employed to determine the stability map of the system and to study the corresponded eigenvalue problem. The related results were used to study the effects of the applied static loads as well as the dimensions of the system on the system. The approach proposed in this paper can be further extended to study dynamic behavior of more complex compliant mechanisms.

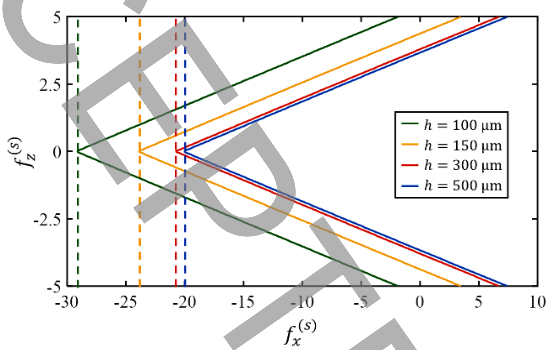

Figure 2: Stability map and the *w*<0.15 limit for a P-flexure assuming *l*=17.6 µm

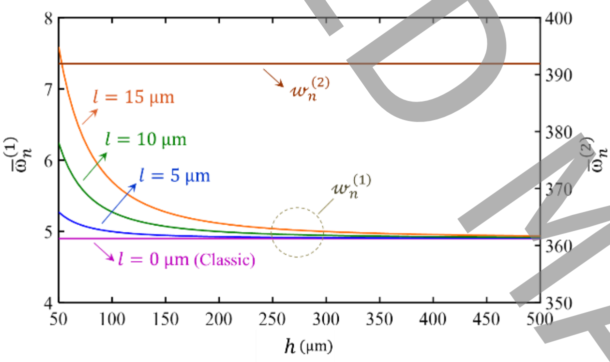

Figure 3: Natural frequencies of a micro-scale P-flexure

4. References

[1] S. Iqbal, R.I. Shakoor, Y. Lai, A.M. Malik, S.A. Bazaz, Experimental evaluation of force and amplification factor of three different variants of flexure based micro displacement amplification mechanism, Microsystem Technologies, 25(7) (2019) 2889-2906.

[2] A. Vafaie, M. Tahmasebipour, Y. Tahmasebipour, A novel capacitive micro-accelerometer made of steel using micro wire electrical discharge machining method, Journal of Micromechanics and Microengineering, 29(12) (2019) 125018.

[3] Y. Gao, C. Yan, H. Huang, T. Yang, G. Tian, D. Xiong, N. Chen, X. Chu, S. Zhong, W. Deng, Microchannel-Confined MXene Based Flexible Piezoresistive Multifunctional Micro-Force Sensor, Advanced Functional Materials, 30(11) (2020) 1909603.

[4] P. Trocha, M. Karpov, D. Ganin, M.H. Pfeiffer, A. Kordts, S. Wolf, J. Krockenberger, P. Marin-Palomo, C. Weimann, S. Randel, Ultrafast optical ranging using microresonator soliton frequency combs, Science, 359(6378) (2018) 887-891.

[5] A. Kravchenko, V. Komenko, W.-J. Fischer, Silicon-On-Nothing Micro-Pirani Gauge for Interior-Pressure Measurement, in: Multidisciplinary Digital Publishing Institute Proceedings, 2018, pp. 1079.

[6] A. Jain, Development of Indigenous Micro-gyroscope Technology, (2019).

[7] S. Afrang, H. Mobki, M. Hassanzadeh, G. Rezazadeh, Design and simulation of a MEMS analog micro-mirror with improved rotation angle, Microsystem Technologies, 25(3) (2019) 1099

[8] S. Awtar, A.H. Slocum, Closed-form nonlinear analysis of beam-based flexure modules, in: ASME 2005 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers Digital Collection, 2005, pp. 101-110.

[9] S. Awtar, S. Sen, A Generalized Constraint Model for Two-Dimensional Beam Flexures: Nonlinear Load-Displacement Formulation, Journal of Mechanical Design, 132(8) (2010).

[10] S. Awtar, S. Sen, A generalized constraint model for two-dimensional beam flexures: Nonlinear strain energy formulation, Journal of mechanical Design, 132(8) (2010).

[11] M. Bakhtiari-Shahri, H. Moeenfard, Topology optimization of fundamental compliant mechanisms using a novel asymmetric beam flexure, International Journal of Mechanical Sciences, 135 (2018) 383-397.

[12] H. Malaeke, H. Moeenfard, A novel flexure beam module with low stiffness loss in compliant mechanisms, Precision Engineering, 48 (2017) 216

[13] F. Ma, G. Chen, Modeling large planar deflections of flexible beams in compliant mechanisms using chained beam-constraint-model, Journal of Mechanisms and Robotics, 8(2) (2016).

[14] M.N. Aghaei, H. Moeenfard, M. Moavenian, Nonlinear extensional-flexural vibrations in variable cross section beams with eccentric intermediate mass, International Journal of Mechanical Sciences, 196 (2021) 106248.

[15] L. Cui, S. Awtar, Experimental validation of complex non-minimum phase zeros in a flexure mechanism, Precision Engineering, (2019) 167-177.

[16] L. Cui, C. Okwudire, S. Awtar, Modeling complex nonminimum phase zeros in flexure mechanisms, Journal of Dynamic Systems, Measurement, and Control, 139(10) (2017).

ACCEPTED MANUSCRIPT