Free vibrations of embedded functionally graded graphene platelets reinforced porous nanocomposite plates with various shapes using p-Ritz method

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ABSTRACT

In this study, the free vibrations of functionally graded graphene platelet-reinforced porous nanocomposite plates with various shapes such as rectangular, elliptical and triangular ones embedded on an elastic foundation are analyzed. To mathematically model the considered plate and elastic foundation, the first-order shear deformation plate theory and Pasternak model are used, respectively. Three types of graphene nanoplatelet distribution patterns and porous dispersion types through the thickness are considered for the nanocomposite plate. To obtain the effective material properties of considered nanocomposite, a micromechanical model is employed. Then, the energy functional of considered functionally graded graphene platelet-reinforced porous nanocomposite plates is expressed and the analytical P-Ritz method is used to solve the vibration problem corresponding to different shapes and boundary conditions, The influences of porosity coefficient, weight fraction of graphene nanoplatelets, elastic foundation coefficients and also the lengths-to-width and -thickness ratios on the natural frequency are analyzed. It is illustrated that the plate with non-uniform and symmetric of first type porosity distribution pattern and the first type graphene nanoplatelets has higher natural frequency. Also, by increasing the porosity coefficient, the natural frequency of the plate associated with all patterns of graphene nanoplatelets is reduced.

KEYWORDS

Free vibration, Porous nanocomposite plates, Plates with various shapes, Elastic foundation, P-Ritz method.

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1. Introduction

In recent years, many studies have been conducted on mechanical behaviors of functionally graded graphene platelets reinforced porous nanocomposite beams, plates and shells. The effects of geometric of nanoplatelets, weight fraction, porosity distribution and geometric parameters on bending, buckling and vibrational behaviors have been investigated [1].

Literature review shows that no study has been performed on free vibration of nanocomposite plate reinforced graphene nanoplatelets with arbitrary shape including rectangular, elliptical and isosceles triangular. In the present study, based on the first order shear deformation theory and using the p-Ritz method, the free vibration of arbitrary-shaped porous nanocomposite plates embedded on elastic foundation is investigated. The elastic foundation is formulated using the Winkler-Pasternak model. Three types of distribution for pores and graphene nanoplatelets through the thickness are considered. The modified Halpin-Tsai micromechanics model and extended rule of mixture are used to determine the effective material properties of the porous nanocomposite.

After convergence study and verifying the accuracy of the present results, a comprehensive parametric investigation is performed to study the influence of the weight fraction and geometric parameters of GPL nanofiller and porosity coefficient on the vibrational behavior of porous nanocomposite plate with various shapes.

2. Problem formulation

In this paper, three types of FG porous plates along with the evenly porosity distribution case, denoted by p_1, p_2, p_3 are considered. To further strengthen the mechanical properties, the metal matrix of the composite plate reinforced by GPLs. And the distribution of GPLs in the metal matrix may be uniform or non-uniform by adjusting the volume fraction along the plate thickness. Three different GPLs patterns are also considered for each porosity distribution which are A, B, C [1].

Three distributions of internal pore inside of the proposed porous plates and three GPL dispersion patterns regarding the varying nanofillers volume contents V_{GPL} across the thickness are assumes.

The variation of Young's module, shear module and mass density through the thickness direction for different porosity distribution can be described by equation (1) and N_0 is the coefficients of porosity.

$$\begin{aligned} E(z) &= E_{\max}\left(1 - N_0\phi(z)\right) \\ G(z) &= G_{\max}\left(1 - N_0\phi(z)\right) \\ \rho(z) &= \rho_{\max}\left(1 - N_m\phi(z)\right) \end{aligned} \tag{1}$$

The effective Young's module and mass density are obtained based on Halpin-Tsai micromechanics model.

The adopted admissible P-Ritz functions which satisfy at least boundary condition for the deflection and rotation of plate are given by equation (4) [2]:

$$\begin{aligned} w(\tau,\xi,\eta) &= \sum_{q=0}^{p}\sum_{i=0}^{q} c_m (2\xi)^i (2\eta)^{q-i} \phi_b^w(\xi,\eta) e^{i\omega\tau} \\ \phi_x(\tau,\xi,\eta) &= \sum_{q=0}^{p}\sum_{i=0}^{q} d_m (2\xi)^i (2\eta)^{q-i} \phi_b^x(\xi,\eta) e^{i\omega\tau} \\ \phi_y(\tau,\xi,\eta) &= \sum_{q=0}^{p}\sum_{i=0}^{q} e_m (2\xi)^i (2\eta)^{q-i} \phi_b^y(\xi,\eta) e^{i\omega\tau} \end{aligned} \tag{2}$$

According to the p-Ritz method, the minimizing of total potential energy with respect to unknow displacement parameters yields:

$$\Pi^* = \frac{\Pi}{\Delta} = \bar{U} + \bar{V}_e - \bar{K} \tag{3}$$

The governing equation for free vibration analysis is expressed as:

$$\begin{aligned} &\left(\left[K\right] - \omega^2\left[M\right]\right)\begin{Bmatrix}\{c\}\\ \{d\}\\ \{e\}\end{Bmatrix} = 0 \\ &\omega = \Omega a\sqrt{\frac{I_{110}}{A_{110}}} \end{aligned} \tag{4}$$

3. Results and Discussion

At the first step, the natural frequencies of elliptical homogenous plate without pore and graphene platelet nanofillers are compared with those given in reference [3], as given in Table 1. An excellent agreement can be found between the provided results and those given in literature.

Table 1. Comparting of natural frequencies of elliptical homogenous plate under simply supported boundary conditions

	[3]		Present	
a/b	$\upsilon = 0/5$	$\upsilon = 0/25$	$\upsilon = 0/5$	$\upsilon = 0/25$
1	5/219	4/865	5/21929	4/865272
1/2	4/442	4/157	4/44171	4/157386

The variation of dimensionless natural frequency of elliptical plate versus to the GPL weight fraction is illustrated in Figure 1 for different porosity and graphene platelets distribution patterns. Fundamental frequency increases by an increase in GPLs weight fraction. Compared to patterns B and C, the effect of GPLs with symmetric pattern A on the natural frequency is more considerable.

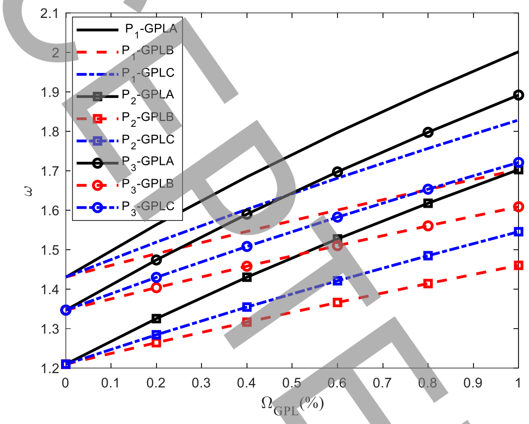

Figure 1. Comparison of natural frequency of elliptical plate versus the GPL weight fraction for the clamped boundary conditions

Figure 2 depicts the variation of dimensionless natural frequency of porous nanocomposite isosceles triangular plate versus the GPL weight fraction for various boundary conditions. Also, Figure 3 illustrates the variations of dimensionless natural frequency of porous nanocomposite rectangular plate versus the GPL shape ratio l_{GPL}/t_{GPL} for various l_{GPL}/w_{GPL}. It can be seen that for higher values of l_{GPL}/t_{GPL}, increasing l_{GPL}/w_{GPL}, the differences between the natural frequencies are negligible.

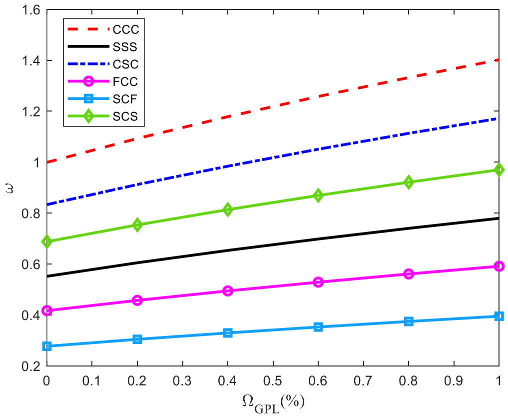

Figure2. Comparison of the natural frequency of isosceles triangular plate versus the GPL weight fraction for various boundary conditions

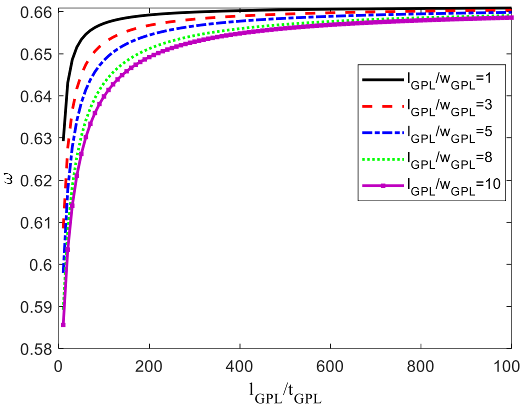

Figure 3. Comparison of the natural frequency of rectangular plate versus the length to thickness ratio of GPLs under CSCS boundary conditions for GPL pattern *A*

4. Conclusions

- The maximum frequencies can be achieved for the no-uniformly symmetric porosity distribution 1 and GPL pattern A.
- An increase in the weight fraction leads to increase of natural frequencies of porous nanocomposite plates.
- Increasing the l_{GPL}/t_{GPL} and l_{GPL}/w_{GPL} ratios result in increasing and decreasing the natural frequencies of porous nanocomposite plates, respectively.

5. References

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