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Solution of the Isotropic Heat Equation Using the Finite Volume Monte Carlo Method

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ABSTRACT: The solution of the heat diffusion equation in most practical applications involving complex geometry, thermophysical properties, and boundary conditions is not simply possible and there are some limitations for available numerical solutions. In this research, the finite volume Monte Carlo method was used for the solution of the isotropic heat equation due to two intrinsic capabilities of the finite volume method; first, each cell is energy conserved and second, the grid transformation is not necessary for complex geometries. The Monte Carlo method is a statistical approach based on the physical simulation of the problem capable to solve heat equation with any degree of complexity. First, a simple problem was investigated for validation of the method by comparing results with the analytical solution. Second, the prediction performance of the finite volume Monte Carlo method was evaluated in a problem with complex geometry, varying properties, and boundary conditions. Finally, the performance of the finite volume Monte Carlo method was investigated in estimating the temperature distribution of a three-layer body with different thermal conductivities and convection boundary condition. In all of the considered test cases, the predicted results were in good agreement with analytical and computational fluid dynamics solutions. It was also indicated that for a relatively small number of particles, it is possible to achieve acceptable accuracy with a low computational cost.

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1. INTRODUCTION

The Monte Carlo method is an efficient approach for the simulation of the conduction heat transfer [1-3]. In most of the practical applications including 3D geometries with arbitrary shaped boundaries, variable thermophysical properties, and complicated boundary conditions using the finite difference scheme in the derivation of the Monte Carlo form of the heat equation is restricted, especially when an unstructured mesh is superposed over the domain. Using the finite volume discretization technique will expand the scope of the Monte Carlo method in the analysis of real-world conduction problems. In the current study, the Finite Volume Monte Carlo (FVMC) method [4] is used in three problems with different levels of complexity to assess its performance under difficult conditions.

2. METHODOLOGY

The FVMC form of the heat equation may be derived by first integrating over a control volume and then applying the Green's theorem and finally using the central difference discretization scheme for the resulting first-order derivatives on each of the cell faces [4]. The final FVMC form of the heat equation may be written as:

$$T_P = F_E T_E + F_W T_W + F_N T_N + F_S T_S + F_T T_T + F_B T_B + S_P \quad (1)$$

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The FVMC method is started by releasing N particles from each point in the solution region and tracing them from cell to cell until they absorbed by one of the domain boundaries. At each step, the random walk direction is determined by generating a uniformly distributed random number, R, and following relations

$$\begin{aligned}
&P \to E \text{ if } 0 < R < F_E \\
&P \to W \text{ if } F_E < R < F_E + F_W \\
&P \to N \text{ if } F_E + F_W < R < F_E + F_W + F_N \\
&P \to S \text{ if } \\
&\quad F_E + F_W + F_N < R < F_E + F_W + F_N + F_S \\
&P \to T \text{ if } \\
&F_E + F_W + F_N + F_S < R < F_E + F_W + F_N + F_S + F_T \\
&P \to B \text{ Otherwise}
\end{aligned} \quad (2)$$

Now the temperature of node P is calculated from:

$$T_P = \frac{1}{N}\sum_{i=1}^{N} T_C(i) + \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{m_i-1} S_P\left(x_j, y_j, z_j\right) \quad (3)$$

3. RESULTS AND DISCUSSION

3.1 Unit cube without heat generation

In this section, the temperature profile on the midline of

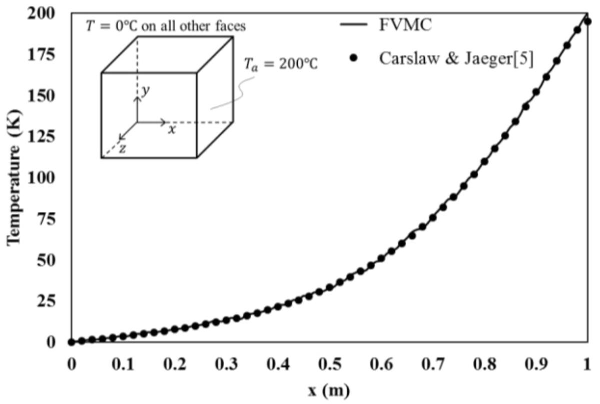

Fig. 1. Comparison of the temperature profiles on the midline of the unit cube

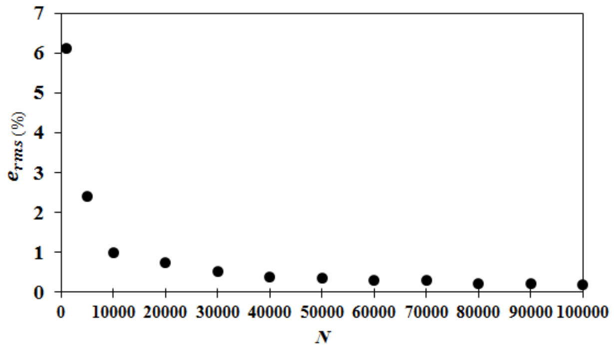

Fig. 2. e_{rms} of the FVMC method on the *z*=0.5 m plane as a function of N

a unit cube with boundary conditions as shown in Fig. 1, is calculated by the FVMC method and the results are compared with the exact data from the Carslaw and Jaeger [5] solution. As shown in Fig. 1, the results are consistent together which confirms the accuracy of the FVMC method.

The relative root mean square error, e_{rms}, of the estimated results on the $z = 0.5$m plane with respect to the total number of investigated particles from each point, N, is plotted in Fig. 2. It is clear from Fig. 2 that by using a relatively small number of particles ($N = 10000$) a very good accuracy is achieved.

3.2 Spherical cavity in a cube with variable k

In order to investigate the robustness of the proposed method to handle problems with complicated geometries, the FVMC method was used to calculate the temperature distribution of a unit cube with a hole inside with a radius of 0.25m. The temperature of the outside surfaces of the cube is assumed zero where a constant heat flux of $q_s'' = 10000$ W/m^2 is applied to the surface of the hole. The thermal conductivity of the medium and the heat source are defined as:

$$k = 10\exp(x^2)\exp(y^2)\exp(z^2) \tag{4}$$

$$g = 100000\cos(\pi x)\cos(\pi y)\cos(\pi z) \tag{5}$$

The temperature distribution on the radial line with an angle of 45 degrees was compared with the computational fluid dynamics (CFD) solution in Figure 3. As it is evident from this Figure, the predicted temperatures from the FVMC method are fully consistent with those from the CFD method.

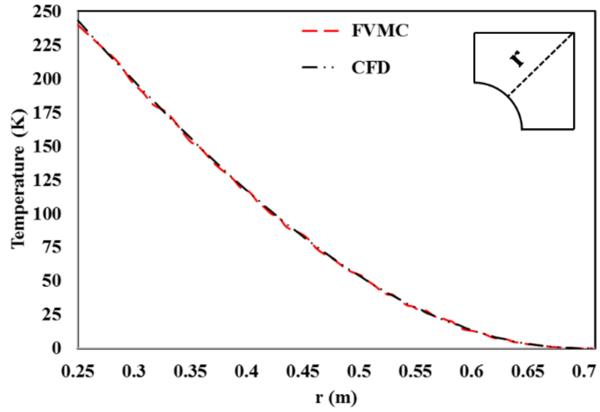

Fig. 3. Comparison of the temperature profiles on the radial line with an angle of 45 degrees

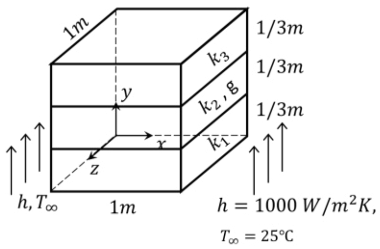

Fig. 4. Geometry and boundary conditions of the three-layered cube

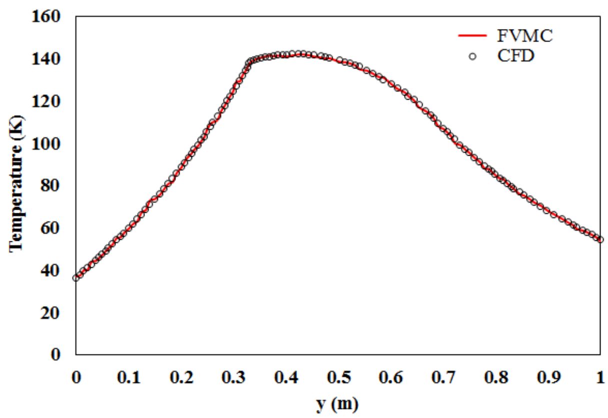

Fig. 5. Comparison of the temperature profiles on the $y = 0.5$ m line

3.3 Three-layered cube

Consider a three-layered cube with different thermal conductivities as $k_1 = 50$ W/mK, $k_2 = 300$ W/mK, and $k_3 = 250$ W/mK where a uniform heat source $g = 500000$ W/m^3 is placed within the middle layer of the body, as shown in Fig. 4. The temperature distribution on the $y = 0.5$ m was compared with the CFD solution in Fig. 5 which are consistent together.

Table 1. e_{rms} **of the FVMC method (%)**

First problem	Second problem	Third problem
0.63	0.82	0.91

4. CONCLUSIONS

Main conclusions of the paper are:

- The FVMC method can predict the temperature distribution in all of the considered test cases with any levels of complexity.
- The calculated e_{rms} for all of the three problems for $N = 50000$ particles are given in Table 1. As evident from this Table, the predictive performance of the FVMC is good even in complicated conditions.
- The FVMC method is quite suitable for the inverse heat conduction problems that only need to calculate the temperature at one or more points.
- It may be better to use the FVMC method in the problems with unstructured meshes that other numerical techniques are incapable of solving the problem.

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