Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 53(2) (2021) 193-196
DOI: 10.22060/mej.2019.16689.6421

2-Dimensional Electroelastic Analysis of Piezoelectric Cylinders Using First-Order Shear Deformation and First-Order Electric Potential Theories

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ABSTRACT: Today, sensors and actuators have a special place in the science and industry world. Therefore, the electromechanical analysis of piezoelectric materials is one of the topics of interest for researchers. To that effect, besides understanding the behavior of piezoelectric structures by optimizing this behavior, they could facilitate designing and manufacturing structures. In this study, using the first-order shear deformation theory and the first-order electrical potential theory and applying the energy method, the governing equations are extracted for thick piezoelectric cylinders subjected to mechanical and electrical loading at their internal and external surfaces under various boundary conditions in two cylinder heads. Then an analytical solution is presented for the governing equations. Using this solution, the results of the electromechanical behavior of the cylinders under different boundary conditions in two cylinder heads are estimated and evaluated. Then these results are compared with the finite element method's results. The analytical solution is not a series solution, and therefore there would be no need to investigate the convergence. Furthermore, it is of less computational volume. The results obtained from the analytical method and the finite element method have a good agreement, showing that the presented analytical solution can be used with higher accuracy.

Review History:

Received: Jul. 04, 2019
Revised: Sep. 14, 2019
Accepted: Nov. 05, 2019
Available Online: Nov. 15, 2019

Keywords:

Thick cylinder
Piezoelectric
Shear deformation theory
First-order electrical potential theory

1. INTRODUCTION

The significance of the electro-elastic analysis of the piezoelectric cylinder lies in its application in a wide range of smart systems such as sensors, actuators, and active controllers [1]. That explains why so many researchers have published papers about the electro-elastic analysis of different kinds of cylinders. Nevertheless, many studies for electro-elastic analysis consider the one-dimensional electro-elastic analysis in a cylinder based on infinite length cylinder assumptions, and a few studies perform the two-dimensional electro-elastic analysis in the cylinder just for special boundary conditions in the cylinder head [2-3].

In this research, the two-dimensional electro-elastic analysis of a piezoelectric cylinder with different boundary conditions in cylinder heads (electrical and mechanical conditions), which is subjected to electro-mechanical loading in its inner and outer surface, is investigated by the analytical solution. Therefore, all electro-mechanical behaviors of the finite length piezoelectric cylinder are investigated according to the first-order shear deformation theory and first-order electrical potential theory. The analytical solution is not a kind of Series solution, hence, it does not need to check the convergence. Furthermore, it is of less computational volume and has less processing time. In order to verify the accuracy of this method, the results are compared with the results of the finite element method analysis, which have good accuracy. Finally, the Two-Dimensional (2D) electro-elastic analysis for the finite length of the piezoelectric cylinder is done, and its electro-mechanical behavior is investigated.

2. GOVERNING EQUATIONS

Fig. 1 shows the cross-section of the piezoelectric cylinder under electromechanical loading on the inner and outer surfaces.

based on the problem is axisymmetric, the functions expressing the electromechanical behavior (displacement field and electric potential) in the cylinder will be only a function of the radial coordinate z and the longitudinal coordinate x, which can be written by Taylor's expansion around the middle layer of the cylinder. Accordingly, we have the function of the electric potential (φ) as follows in the cylinder.

$$\varphi = \varphi(z,x) = \varphi(0,x) + z\left.\frac{\partial \varphi(z,x)}{\partial z}\right|_{z=0} + \frac{z}{2!}\left.\frac{\partial^2 \varphi(z,x)}{\partial z^2}\right|_{z=0} + \cdots = \varphi^0 + z\,\varphi^1 \quad (1)$$

Using the first-order shear deformation theory, the first-order electric potential theory, the definition of mechanical-electrical resultants [4-5], and applying the energy method [6], the system of governing equations and boundary conditions will be derived as follow:

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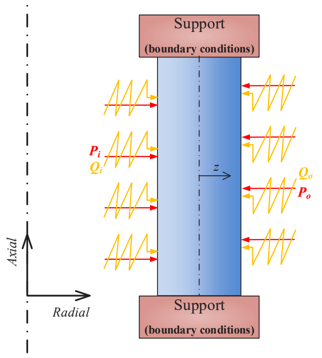

Fig. 1. Cross-section of the piezoelectric cylinder under electro-mechanical loading in inner and outer surfaces

$$
\begin{cases}
R\dfrac{\mathrm{d}N_x^m}{\mathrm{d}x}=0 \\
R\left(Q_x^m-\dfrac{\mathrm{d}M_x^m}{\mathrm{d}x}\right)=0 \\
R\left(\dfrac{N_\theta^m}{R}-\dfrac{\mathrm{d}Q_x^m}{\mathrm{d}x}\right)=P_i\left(R-\dfrac{h}{2}\right)-P_o\left(R+\dfrac{h}{2}\right) \\
\dfrac{M_\theta^m}{R}+N_z^m-\dfrac{\mathrm{d}M_{xz}^m}{\mathrm{d}x}=\dfrac{h}{2R}\left(P_i\left(\dfrac{h}{2}-R\right)-P_o\left(R+\dfrac{h}{2}\right)\right) \\
R\dfrac{\mathrm{d}N_x^e}{\mathrm{d}x}=Q_i\left(R-\dfrac{h}{2}\right)-Q_o\left(R+\dfrac{h}{2}\right) \\
\left(N_z^e-\dfrac{\mathrm{d}M_x^e}{\mathrm{d}x}\right)=\dfrac{h}{2R}\left(Q_i\left(R-\dfrac{h}{2}\right)-Q_o\left(R+\dfrac{h}{2}\right)\right)
\end{cases} \tag{2}
$$

$$
\begin{aligned}
&[N_x^m\delta U_x^0+M_x^m\delta U_x^1+Q_x^m\delta U_z^0+M_{zx}^m\delta U_z^1 \\
&+N_x^e\delta\varphi^0+M_x^e\delta\varphi^1]\Big|_{0,L}=0
\end{aligned} \tag{3}
$$

3. RESULT AND DISCUSSION

To study the electro-mechanical behavior, a cylinder with a middle layer diameter of 50 mm, a thickness of 20 mm and a length of 800 mm is considered. This piezoelectric cylinder is made PZT-4 [1]. The inner layer of the cylinder has 15×10^{-7} C/m^2 surface charge density and the outer layer is subjected to 600 kPa pressure. The cylinder is studied in two states (1) two clamped heads with zero electric potential, and (2) one head with zero electric potential and another one with zero surface charge density. The average total CPU time taken to perform analytical resolution is 9.586 seconds by Maple 13 software.

By comparing the state (1) and (2) in Fig. 1, it can be realized when the cylinder head is not constrained, leading to an increase in the maximum axial displacement value. As can be seen in Fig. 2 too; there is good agreement between the analytical method and the finite element method.

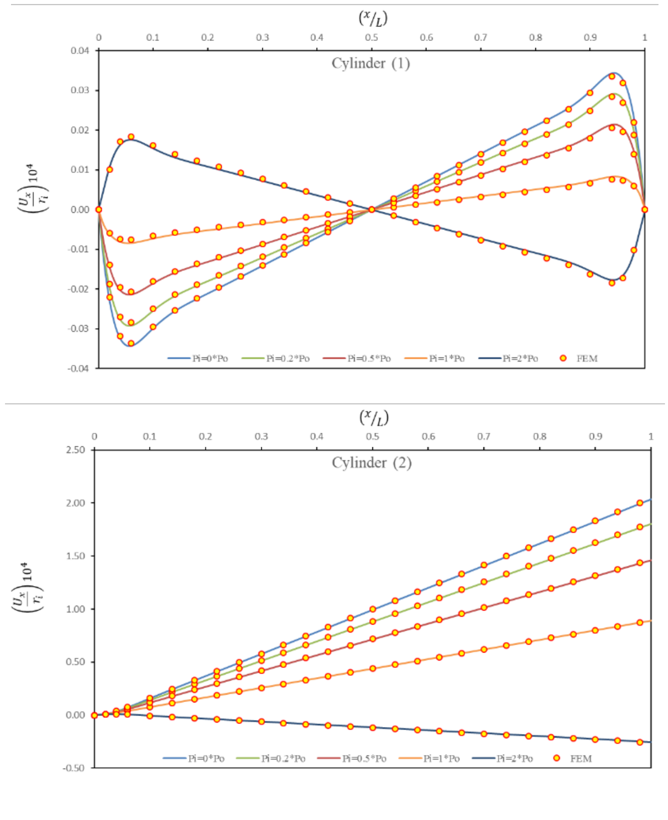

Fig. 2. Distribution of axial displacement in the middle layer of piezoelectric cylinders with various boundary conditions

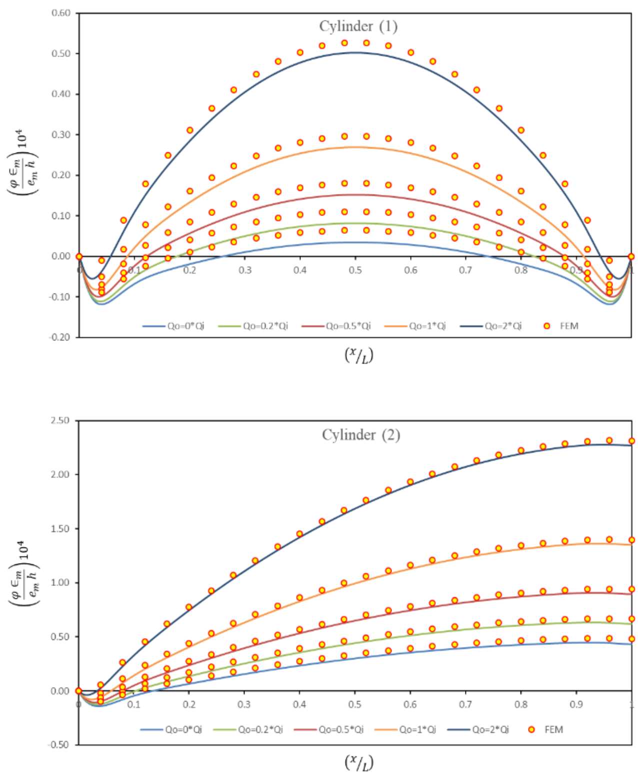

Fig. 3. Distribution of electrical potential in the middle layer of piezoelectric cylinders with various boundary conditions

4. CONCLUSIONS

Some of the important achievements of this research are listed as following:

The results show that, unlike the displacement field, it does not have much variation in the thickness direction of the cylinder. The electrical potential, even in areas far off the boundaries, changes in thickness; therefore, a two-dimensional mechanical-electrical analysis of piezoelectric actuators, sensors, and controllers of finite length cylinders is of particular importance.

Evaluation of the results of the cylinders with different boundary conditions in the two heads of cylinder shows that the absolute maximum value of the mechanical and electrical behavioral parameters of the cylinders (displacement field, electric potential, and stresses) occurs near the boundaries or at the cylinder boundaries; so, the designers should pay particular attention to border areas.

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HOW TO CITE THIS ARTICLE

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