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# Unstable Aircraft Flight Control Based on Linear Matrix Inequality with Consideration of Control and Saturation Constraints

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**ABSTRACT:** In this paper, limitation in actuator capacity has been used as a key role in the design of the flight control system. In order to guarantee the performance and stability of flight control systems in the presence of saturation, in flying high angle of attack area, the development of linear matrix inequality, optimization techniques, and numerical methods are proposed. Also, in this paper, the combination of two anti-windup methods and the direct saturation method in the tracking problem of the flight path angle is discussed. For this purpose, the nonlinear model of the aircraft is modeled, moreover the linear model is obtained at the trim operation conditions. Then the controller is designed to track flight path angle maneuver regardless of saturation. In the following, considering the maximum disturbance involved in aircraft maneuvering, a safe controller that guarantees performance and stability is designed, and the gain scheduling technique to prevent conservatism in the use of controllers is applied. The results of the nonlinear and linear model of the aircraft are presented in tracking flight path angle at a high angle of attack with consideration of control and saturation constraints in unstable operation conditions. Simulation results indicate the improvement of the mentioned control method for an unstable aircraft.

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## 1. INTRODUCTION

One of the essential issues in the design of flight control systems, especially in flight at high angles of attack, is saturation. From flight applications at high angles of attack can be used to reduce the landing distance, terrain following, and terrain avoidance and escape maneuvers. Obviously, the features will be compromised by the actuator saturation. In addition, the more intense the application levels of disturbance with saturation leads to catastrophic accidents. Aviation accidents related to saturation in the actuator of control surfaces are mentioned in reference [1].

To prevent catastrophic events and resolve this problem, since the mid-1990s, many solutions have been developed to solve windup problems with the high-performance guarantee requiring high-cost simulations and tests to ensure stability and performance. Many of these methods were based on the control signal that was used for discrete control systems with time. The other group, which is mostly used for continuous-time and is definitely effective in guaranteeing performance and stability, is Linear Matrix Inequality (LMI) [2]. In recent years, LMI methods, optimization techniques, and numerical methods have been used to integrate controllers with saturation problems, resulting in improved performance and stability. Saturation exposure techniques are included in two parts. The first class is named Anti-Windup (AW), which is usually added to the controller and saturation is not considered in its design. The AW control block is activated once the nominal controller is saturated. The second category is known as the direct method [3, 4].

The proposed method in this paper is based on the combination of two basic techniques, that's mean the combination of the AW method and the direct method, which has the ability to apply unstable systems. Along with its application in unstable systems, the method combines the use of a scheduled controller to reduce the conservatism designed [5, 6]. In this paper, the behavior of unstable aircraft in tracking the flight path angle in the existence of AW controller with gain scheduling based on the disturbance is investigated. To do this, the nominal controller is initially considered without considering the saturation range for each maneuver, then for maximum disturbance, the scheduled controller gain using the LMI on the anti-windup controller to ensure the performance and stability of the linear and nonlinear model applies. For a numerical example of an aircraft, simulation has been performed and the results indicate the effect of the new control method on applying unstable models to the aircraft's longitudinal dynamic behavior in tracking maneuver of the flight path.

## 2. DESIGNING CONTROLLER BASED ON LINEAR MATRIX INEQUALITY

One of the most important and most dangerous nonlinear factors in any system is related to the actuators saturation, various studies have been presented to illustrate the problems. Failure to consider this factor can significantly reduce system performance or at worst case the system to become unstable even for a stable open loop. Saturation usually happens in real-world situations, since actuators have an inherently

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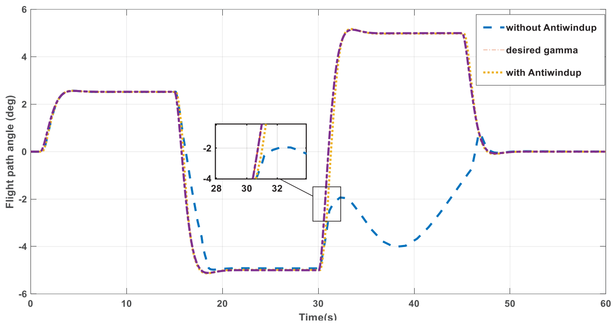


**Fig. 1. The response of the nonlinear model of the aircraft to the flight path angle tracking maneuver with and without anti-windup**

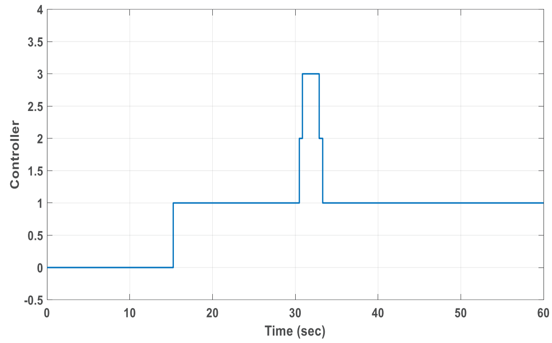


**Fig. 2. scheduling controller designed for the nonlinear model of the aircraft** $K_{formal} = 0, K_1 = 1, K_2 = 2, K_3 = 3, K_{safe} = 4$

limited capacity, so they can't always do what the ask [3].

For disturbance, different boundaries are considered and the assumption is that the maximum value of the disturbance is known (Eq. (1)), by using the gain scheduling method, the design of the unit controller between the maximum and minimum disturbance, which represents the conservative behavior, is prevented.

$$\omega^T(t)\omega(t) \le \omega_{\max}^2 \tag{1}$$

In the following, three basic constraints, including invariant set, constraints, and performance, will be presented in the form of LMI to take into account saturation nonlinearity when designing the controller in the presence of disturbances. To do this, we use the Lyapunov function, in this paper $P$ and $x$ are respectively positive constant matrix and state vector of the system [7].

For the first constraint, based on the invariant set of LMI, the goal is to remain within the constant setting despite the acts of disturbance, which is expressed as inequality (Eq. (2)) [8].

$$\begin{pmatrix} A^T P + PA + K^T B_2^T P + PB_2 K + \alpha P & PB_1 \\ * & -\alpha I \end{pmatrix} < 0 \tag{2}$$

Inequality in Eq.(3) emphasizes the unsaturation of a controller designed for the second constraint of linear inertia inequality, $\varepsilon(P, \omega_{\max})$ defined by an invariant set [2, 4].

$$\begin{pmatrix} Q & F^T \\ * & \dfrac{u_{\lim}^2}{\omega_{\max}^2} \end{pmatrix} > 0 \tag{3}$$

In the performance approach based on the LMI method, in addition to the fact that the controller within the defined region should not be saturated, it should provide the premium feasible answer for the disturbance. In this paper, gains $H_\infty$ or $L_2$ have been used as system performance. The basic idea is finding the smallest $\Gamma$ in inequality Eq. (4)[9].

$$\begin{pmatrix} QA^T + AQ + B_2 F + F^T B_2^T & B_1 & QC^T + F^T D_2^T \\ * & -\Gamma I & D_1^T \\ * & * & -\Gamma I \end{pmatrix} < 0 \tag{4}$$

## 3. RESULTS AND DISCUSSION

Simulation results for nonlinear and linear models in unsteady working conditions, in the presence of a nominal controller and anti-windup controller, are presented. According to the results presented, for the nonlinear model and the linear model, it can be seen that the nominal controller is not capable of tolerating the disturbance and is saturated (Fig. 1). Also, the role of a safe controller is visible in the stability and performance which is not saturated, while the nominal controller is saturated and has lost performance and stability. The results are presented for disturbance in the reference input of the flight path angle. Also, the controllers used at any given time for the nonlinear model is presented in Fig. 2.

## 4. CONCLUSION

Saturation of actuators is one of the significant subjects in aircraft flight control problems. Most of the proposed methods are to solve this problem for stable systems. The results showed the significant and effective performance of the gain scheduling in improving the performance and stability of the linear and nonlinear unstable model of the aircraft in maneuver tracking the flight path angle. It was observed that in the presence of a nominal controller, saturation occurs and stability and performance are lost. By adding a safe controller, stability improvements and performance were achieved. Also, the gain scheduling method was used to prevent the conservatism of the designed controllers. saturated controller graphs with time, which indicates the online activity of the controllers, reflects the number of controllers, between the maximum and minimum disturbances, and improved performance and stability by choosing a higher gain controller.

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