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Modeling and trajectory tracking control of non-holonomic mobile robot with revolute-prismatic joints

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ABSTRACT: One of the main topics in the field of robotics is the modeling and control of mobile robots in the trajectory tracking problem. In this paper, the kinematic and dynamic models of a manipulator connected by revolute-prismatic joints and installed in a non-holonomic wheeled mobile platform are first derived by applying the recursive Gibbs-Appell method. Indeed, by employing this dynamic methodology, one gets rid of the difficulties of Lagrange Multipliers that originate from non-holonomic constraints. Then, a nonlinear predictive approach is applied to design the kinematic and dynamic control laws to generate trajectory tracking control commands of the non-holonomic robot. In this method, the nonlinear responses of the mobile robot are predicted using the Taylor series. The optimal control laws are analytically developed by minimizing the difference between the predicted and the desired responses of the system outputs. The obtained control inputs from a multivariable kinematic controller in the first stage are then used as the desired values to be tracked by the dynamic controller. Finally, the results of numerical simulations are then presented to emphasize the ability of the proposed method in the mathematical modeling and simultaneous trajectory tracking control of the mobile base and end-effector of such complex robotic systems.

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1. INTRODUCTION

Moving manipulators typically comprise underactuated systems under non-holonomic constraints. The main specification of underactuated systems is that for them the number of states that are to be controlled is higher than the number of control inputs. Also, the dynamics of coupling between a moving platform and manipulator arm, and the existing nonlinearities and model uncertainties create many control challenges for the trajectory tracking of moving manipulators. Diverse techniques have been presented by the researchers to deal with these problems, including the use of sliding mode control [1], robust control [2], fuzzy control [3], adaptive control [4], and neural network control [5]. However, the results of this work are limited to a moving platform only and it does not reflect the coupling effects arising from the installation of a mechanical arm on it.

2. SYSTEM KINEMATICS

This paragraph presents the kinematics of a multi-rigid-link robotic manipulator with R-S joints which is installed on a moving base. Each link's coordinate system ($x_i y_i z_i$) is oriented based on the laws proposed by Denavit & Hartenberg (D-H). The frame attached to the moving platform is $x_0 y_0 z_0$, whose origin is fixed at Point P; the x_0 axis is along the axis of symmetry of the moving base, y_0 is along the rolling wheels' axis of rotation (toward the left rolling wheel), and the z_0 axis completes the orthogonal coordinate system. Also, the ground-fixed $X_G Y_G Z_G$ frame can be taken as the global reference frame.

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To accomplish the mathematical modeling of the above robotic system, the succeeding assumptions are adopted: 1) the wheels roll on an even surface without slipping, 2) The moving base is confined to travel in the $X_G O_G Y_G$ plane, and 3) To uphold the no-slipping condition, Point P's velocity is assumed to be co-linear with the platform's axis of symmetry.

3. SYSTEM DYNAMICS

The G-A formulation uses the Gibbs function (acceleration energy) to get the motion equations of a mechanical system. For this reason, a set of independent quasi-velocities is selected. These are quantities that are linear combinations of the generalized velocities but which cannot be integrated into the generalized coordinates. After constructing the Gibbs function in terms of accelerations, we calculate the differentiations of this function with respect to the selected quasi-accelerations. Finally, by equating the obtained result to the generalized forces associated with the quasi-velocities, the governing motion equations are derived. It can be easily proved that the Gibbs function for a rigid body has the following form

$$S=\frac{1}{2}m\vec{a}_G^T\cdot\vec{a}_G+\frac{1}{2}\dot{\vec{\omega}}^T\cdot I_G\,\dot{\vec{\omega}}+\dot{\vec{\omega}}^T\cdot\tilde{\omega}\,I_G\,\vec{\omega} \qquad (1)$$

m and I_G respectively denote the mass and moment of inertia about the centroid, $\vec{a}_G$ is the centroid's acceleration,

$\vec{\omega}$ and $\dot{\vec{\omega}}$ respectively represent the rigid body's angular velocity and acceleration, and $\tilde{\omega}$ indicates the skew-symmetric matrix associated with vector $\vec{\omega}$.

The inverse dynamic motion equations of the examined moving robotic system are obtained as

$$\frac{\partial S}{\partial \dot{v}_A} = \frac{\partial U}{\partial v_A} \tag{2}$$

$$\frac{\partial S}{\partial \ddot{\theta}_i} = \frac{\partial U}{\partial \dot{\theta}_i} \quad j = 0,1 \tag{3}$$

$$\frac{\partial S}{\partial \ddot{\eta}} = \frac{\partial U}{\partial \dot{\eta}} \tag{4}$$

where U are the generalized forces associated with quasi-velocities For control purposes, it is desired to group the coefficients of quasi-accelerations on the left-hand side of the equations and to arrange the remaining dynamic effects on the right-hand side. By organizing these differential equations in a matrix format, the direct dynamic motion equations are derived as

$$I(\vec{\Theta})\ddot{\vec{\Theta}} = \vec{\mathrm{Re}}(\vec{\Theta},\dot{\vec{\Theta}}) + \vec{\tau} \tag{5}$$

4. PROBLEM STATEMENT AND CONTROL SYSTEM DESIGN

The main task of the suggested control system is to determine the input torques that are applied to the respective rolling wheels and the rotary manipulator joint and to obtain the input force exerted on the sliding joint of the manipulator so that the actual robot's platform and end-effector can follow the trajectory of the reference robot's platform and end-effector. There are two phases in the suggested control system. In the initial phase, a kinematic controller is designed to find the proper input velocities that can converge the position errors of the system to zero. Then, a dynamic controller is designed to derive the control torques and forces that can converge the robot's angular and linear velocities to the desired linear and angular velocity inputs obtained from the kinematic controller.

5. TRACKING ERROR FORMULATION

To design a kinematic controller for the mentioned robotic system, the tracking error of the entire system (including the traveling base and the link of the manipulator) should be evaluated as

$$\dot{x}_{P_e} = v_{P_r} \cos\theta_{0,e} - v_{P_a} + y_{P_e}\dot{\theta}_{0,a} \tag{6}$$

$$\dot{y}_{P_e} = v_{P_r} \sin\theta_{0,e} - x_{P_e}\dot{\theta}_{0,a} \tag{7}$$

$$\dot{\theta}_{0,e} = \dot{\theta}_{0,r} - \dot{\theta}_{0,a} \tag{8}$$

$$\dot{\theta}_{1,e} = \dot{\theta}_{0,e} + \left(\dot{\theta}_{1,r} - \dot{\theta}_{1,a}\right) \tag{9}$$

$$\dot{\eta}_e = \dot{\eta}_r - \dot{\eta}_a \tag{10}$$

In the next section, we use the tracking error dynamics (Eqs. (6) through (10)) to design the kinematic controller.

6. KINEMATIC CONTROL DESIGN

Here, a novel kinematic controller will be developed by employing the predictive control scheme. For this purpose, Eqs. (6) through (10) are expressed in the format of state space as

$$\dot{x}_1 = v_{P_r} \cos x_3 - u_1 + x_2 u_2 \tag{11}$$

$$\dot{x}_2 = v_{P_r} \sin x_3 - x_1 u_2 \tag{12}$$

$$\dot{x}_3 = \dot{\theta}_{0,r} - u_2 \tag{13}$$

$$\dot{x}_4 = \dot{x}_3 + \left(\dot{\theta}_{1,r} - u_3\right) \tag{14}$$

$$\dot{x}_5 = \dot{\eta}_r - u_4 \tag{15}$$

The purpose of this section is to stabilize the underactuated robotic system described by Eqs. (11) through (15) by finding proper control laws. Hence, according to the state-space model of the considered system, these output functions can be written as

$$y_1 = x_1 - \beta\, x_2\, \mathrm{sgn}\left(\dot{\theta}_{0r}\right) \tag{16}$$

$$y_i = x_{i+1} \quad i = 2,\ldots,4 \tag{17}$$

In this method, system outputs for subsequent time steps are initially predicted using Taylor series expansion and then the current control inputs are found by continuously minimizing the predicted tracking errors. In this respect, for stabilization problems, a quadratic point-wise objective function can be defined as

$$J(\vec{u}) = \frac{1}{2}\sum_{i=1}^{4} w_i\, y_i^2(t+h) \tag{18}$$

The predicted output response in subsequent time steps is approximated by the Taylor series expansion at time *t*, as follows:

$$y_i(t+h) = y_i(t) + h\,\dot{y}_i(t) \quad i = 1,\ldots,4 \tag{19}$$

The output equations and their derivatives are substituted into Eq. (19) and then inserted into Eq. (18) to acquire the expanded objective function as a function of control inputs. The optimal control laws for $u_i(t)$, are then obtained by minimizing the objective function. So, the control inputs are derived by applying the necessary optimality condition:

$$\frac{\partial J}{\partial u_i} = 0; \quad i = 1,\ldots,4 \tag{20}$$

7. DEVELOPMENT OF THE DYNAMIC CONTROL LAWS

Here, a nonlinear prediction-based controller is developed by employing the robotic manipulator's dynamic models. In state-space format, this system's dynamic model (expressed by Eq. (5)) can be represented as

$$\dot{x}_i = f_i(\vec{x}) + U_i \quad i = 1,\ldots,4 \tag{21}$$

The design of the predictive controller for the dynamic model is similar to that of the kinematic model. The objective here is to maintain the system outputs close to desired responses achieved in the previous section. Here again, a point-wise objective function that minimizes the tracking error for subsequent time steps is presented as

$$J_1(\vec{U}) = \frac{1}{2}\sum_{i=1}^{4} w_i' e_i^2(t + h_1) \tag{22}$$

Where

$$e_i(t + h_1) = y_i(t + h_1) - y_{i,d}(t + h_1) \quad i = 1 \ldots 4 \tag{23}$$

Performing a first-order Taylor series expansion is enough to obtain y_i, and their desired values (i.e., $y_{i,d}$).

$$y_{i\ o\ i,d}(t + h) = y_{i\ o\ i,d}(t) + h_1 \dot{y}_{i\ o\ i,d}(t) \quad i = 1,\cdots,4 \tag{24}$$

Substituting Eq. (24) into Eq. (23) and subsequently into the performance index present by Eq. (22) and using the state-space form of the dynamic motion equations (Eq. (21)) yields

$$J_1(\vec{U}) = \frac{1}{2}\sum_{i=1}^{4} w_i' \left[(x_i - x_{i,d}) + h_1\left(f_i + U_i - \dot{x}_{i,d}\right)\right]^2 \tag{25}$$

Now, by enforcing the optimality condition, the optimal control rules can be extracted as

$$U_i = -\frac{1}{h_1}\left[e_i + h_1\left(f_i - \dot{x}_{i,d}\right)\right] \quad i = 1,\cdots,4 \tag{26}$$

8. CONCLUSION

In this article, the motion equations of a single link movable robotic manipulator with R-S joints are extracted in closed-form. The dynamic model is based on the G-A methodology to derive the motion equations of the mentioned robotic system. Then, by using the predictive control approach, a kinematic controller has been optimally and recursively designed to get the preferred angular and linear velocities of the movable platform and manipulator links so that their position error dynamics converge to zero. Furthermore, a dynamic controller has also been analytically and symbolically developed to track the desired velocities obtained from the kinematic controller and also to find the proper dynamic control laws in the form of input control torques and forces.

REFERENCES

[1] S. Yi, J. Zhai, Adaptive second-order fast nonsingular terminal sliding mode control for robotic manipulators, ISA Transactions, 90 (2019) 41-51.

[2] M. Boukens, A. Boukabou, M. Chadli, Robust adaptive neural network-based trajectory tracking control approach for nonholonomic electrically driven mobile robots, Robotics and Autonomous Systems, 92 (2017) 30-40.

[3] S. G. Tzafestas, K. M. Deliparaschos, G. P. Moustris, Fuzzy logic path tracking control for autonomous non-holonomic mobile robots: Design of System on a Chip, Robotics and Autonomous Systems, 58(8) (2010) 1017-1027.

[4] L. Xin, Q. Wang, J. She, Y. Li, Robust adaptive tracking control of wheeled mobile robot, Robotics and Autonomous Systems, 78 (2016) 36-48.

[5] G. Yi, J. Mao, Y. Wang, S. Guo, Z. Miao, Adaptive tracking control of nonholonomic mobile manipulators using recurrent neural networks, International Journal of Control, Automation and Systems, 16(3) (2018) 1390-

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