



## Free vibration of axially functionally graded nanobeam with an attached mass based on nonlocal strain gradient theory via new ADM numerical method

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**ABSTRACT:** The purpose of this paper is to study the free vibrations along a longitudinal line of a targeted graded nano beam with an attached mass based on the theory of nonlocal strain gradient theory using the asymptotic development method. nowadays due to the importance and usage of nano structures, investigation and recognition of their mechanical and mechanical properties seem necessary. Due to its small size and behavior depending on its size and the inability of classical theories to predict dependant mechanical behaviors, non-classical theories have been used. The governing equations and boundary condition relations of targeted graded nano beam with five boundary conditions simple simple, clamped-free, clamped-clamped, clamped simple, and clamped-attached mass have derived by using Hamiltonian principle. Then differential equation is solved analytically to obtain the frequency equations for the five boundary conditions. In the following, dimensionless frequencies by using the solving zero and the first order of numerical asymptotic development method have derived. Advantage of this method is simplicity, noncomplex mathematical relations and proper coding execution time. Eventually, the effect of nonlocal parameters, material length scale, type of material, the ratio of length to thickness, and mode number on free vibration nano beam has been investigated.

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## 1. INTRODUCTION

Nonlocal elasticity theory is presented by Eringen [1]. In this theory, the stress at a point is dependent on the strain at all points. Strain gradient theory was first presented by Mindlin [2]. In elastic gradient elasticity theory, strain gradient is used to express the effect of four measured parameters. The modified strain gradient theory is presented by Lam et al. [3]. In this theory, the effect of three different length scales is used to express the effect of the effect. Couple stress theory is presented by Toupin [4].

Cao et al. [5], studied the free vibration of an axially functionally graded beam with different boundary conditions and compared the results with finite element results and the results of other literature, in this paper, the effect of the gradient parameter on three natural frequencies is discussed.

## 2. METHODOLOGY

The purpose of this study is to investigate the free vibration of a nano beam functionally graded nano beam with concentrated mass using strain gradient theory. The solution of this paper will be done using the new method of approximate development. After extracting the differential equations of the beam and then nondimensionalization of equations, the first frequencies for boundary conditions are extracted from the solution of the frequency equation. In the following, the displacement equation is extended as a series that is multiplied by a small perturbation parameter. This method consists of two order solving (zero and one order)

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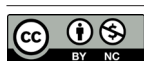
solving, which is extended to one sentence to solve the zero frequency of frequencies and is extended to solve the order of one to two sentences. Finally, the dimensionless equations are solved by solving the differential equations

## 3. RESULTS AND DISCUSSION

In this section, we use the proposed method for free vibration analysis of a functionally graded nano beam with different boundary conditions. In this paper, nano beam is considered from aluminum and titanium. It is the right side of the pure titanium and the left side of the pure aluminum. The nano beam dimensions are  $\frac{L}{H} = 10, B = 2H$ .

In Fig. 2, the effect of size-dependent parameters on the first dimensionless natural frequencies of a functional nano beam for the simply boundary condition is plotted.

Increasing the dimensionless nonlocal parameter decreases the dimensionless natural frequencies as well as increasing the dimensionless longitudinal parameter, which increases the frequency values due to the nano beam stiffness changes due to the effect of size on the nano beam. It is also seen in Fig. 2 that the dimensionless natural frequencies can be larger or smaller than the classical model, depending on the different values of the nonlocal parameter and the longitudinal scale parameter of the material. Classical theories are not suitable for modeling nano beam and must consider the nonlocal effects in the static and dynamic analysis of these structures. The effect of the small-scale parameter of the nano beam makes the nanotube behavior closer to the softer nano beam.



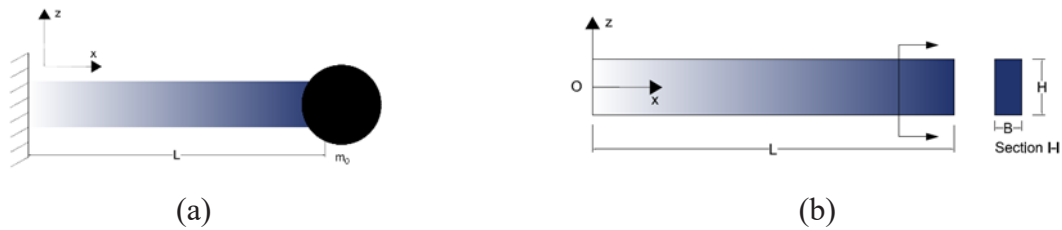


Fig. 1. The geometry and coordinate system of an FG nano beam (a) with attached mass (b) without attached mass

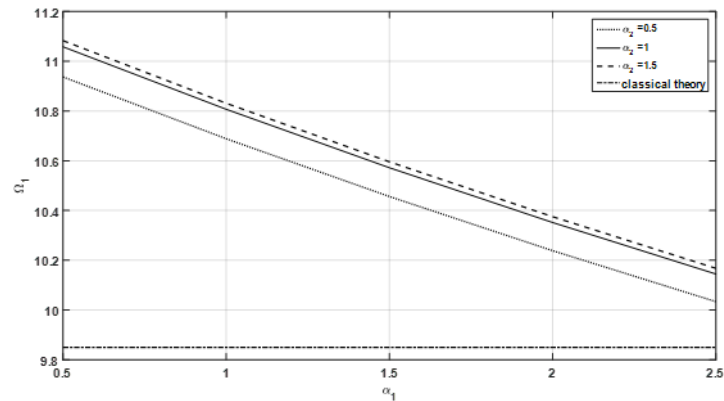


Fig. 2. The first order of dimensionless natural frequencies varying with the size-dependent parameters  $\alpha_1$  and  $\alpha_2$  for simply supported FG beams ( $k=1$ )

#### 4. CONCLUSION

- As the power index parameter ( $k$ ) increases, the natural frequencies increase, indicating the influence of the power index parameter on the natural frequencies. As the nanotube changes, Young's modulus increases and the mass density decreases and also increases the natural frequency and the non-local effects on the frequency response.

-Longitudinal free vibration analysis revealed that different length-thickness ratios did not affect the values of natural frequencies.

-The dimensionless natural frequencies decreased with increasing nonlocal parameter ( $ea$ ) and increased with increasing length scale parameter ( $l$ ).

-The natural frequencies of the nonlocal strain gradient model, depending on the values of the size-dependent parameters, can be larger or smaller than the classical model. By increasing the parameter length of the material scale as well as decreasing the nonlocal parameter, it results in increased scaling behavior of the functional nano beam.

-The use of strain gradient theory in nano and micro scale is considered necessary. As the size is smaller, the linear frequency ratio of these theories leads to more classical theory.

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