Control of heterogeneous traffic flows in presence of pocket loss, time-varying communication delay and actuator lag

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ABSTRACT: This paper deals with the control design and internal and string stability analysis of heterogeneous traffic flows with bi-directional communication topology under random data loss, time-varying communication delay, and actuator lag. A third-order linear model is employed to describe the longitudinal dynamics of each vehicle and the constant spacing policy is employed to adjust the inter-vehicle spacing. In the practical implementation of vehicular networks, due to the high amount of different exchanged information between vehicles and infrastructures, data loss and communication delay are unavoidable effects that may cause adverse effects on the closed-loop performance. Moreover, the actuator lag is an inherent characteristic of the engine which causes delay in implementing the control commands. Therefore, all these issues are considered in system modeling and stability analysis, simultaneously. A linear control protocol using the relative position and velocity measurements with respect to predecessor and subsequent vehicles is introduced for each following vehicle. The Lyapunov-Krassovskii theorem is employed to derive the necessary conditions on control parameters assuring internal stability. Afterward, by performing the error propagation analysis in the frequency domain, sufficient conditions on control parameters assuring string stability are obtained. Finally, several simulation results are provided to show the effectiveness of the presented algorithm.

1- Introduction

Traffic congestion has numerous adverse significances such as air and noise pollution, growing fuel costs and traveling time and decreasing safety, etc. [1]. Vehicular platooning is a useful tool to achieve an Intelligent Transportation System (ITS). In an autonomous vehicle, the upper-level control according to vehicle position in platoon and leader acceleration calculates the desired acceleration of each following vehicle to track the lead vehicle velocity [2, 3].

Two major strategies are used to control inter-vehicle spacing. In the constant spacing strategy, the inter-vehicle spacing is always constant while in the constant time gap strategy the traveling time between vehicles is constant [4, 5].

A convoy is called internal stable if the closed-loop dynamics be asymptotically stable. Moreover, if the amplitude of spacing error of consecutive vehicles has a decreasing trend, the convoy is string stable [6]. In an ITS, due to the huge amount of data transmission, some unwanted effects such as communication delay and data loss are the source of instability. In previous works, the simultaneous effects of data loss, time delay, and actuator lag have not been considered on bi-directional networks.

In this paper, a third-order linear model is employed to describe the longitudinal dynamics of each following vehicle. By considering random data loss, time-varying delay, and actuator lag, and based on the bi-directional topology, a consensus law is defined for all vehicles. By utilizing the Lyapunov-Krasovskii theorem, sufficient conditions guaranteeing internal stability are derived. After that, necessary conditions on control parameters assuring string stability will be obtained through the analysis in the frequency domain. The main novelties of this research are described as 1) analyzing the internal stability of bi-directional vehicle convoys by considering random data loss, time-varying delay, and actuator lag. 2) String stability in presence of aforementioned effects and 3) decoupling the large-scale traffic flow to an individual third-order dynamical model which simplifies the stability analysis dramatically.
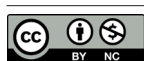
2- System Modeling and Control Design

Consider a convoy of vehicles consisting of a leader and N followers as depicted in Fig. 1. The longitudinal dynamics of a vehicle is described as follows [7]:

$$\dot{\mathbf{x}}_i = \mathbf{A}_1\mathbf{x}_i + \mathbf{A}_2 u_i, \quad \mathbf{x}_i = [x_i, v_i, a_i]^T \qquad (1)$$

where $\mathbf{A}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\tau \end{pmatrix}, \mathbf{A}_2 = [0,0,1/\tau]^T$. Also, the

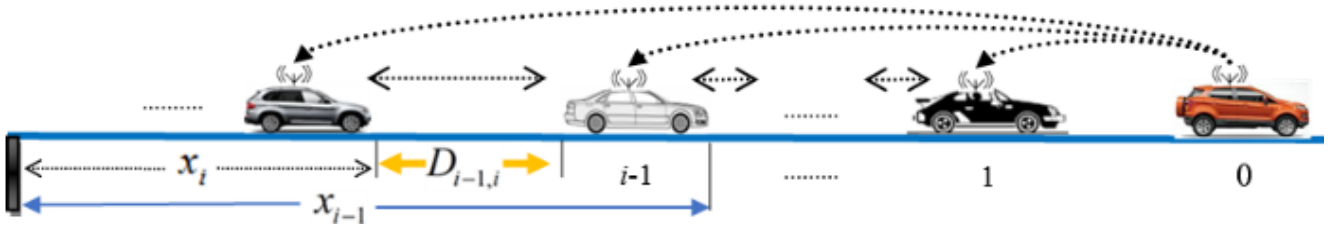*Corresponding author's email: h.chehardoli@abru.ac.ir

Fig. 1. A vehicle convoy

leader dynamics is described as follows:

$$\dot{\mathbf{x}}_0 = \mathbf{A}_0 \mathbf{x}_0, \quad \mathbf{x}_0 = [x_0, v_0, a_0]^T, \mathbf{A}_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

The spacing error of i-th vehicle is defined as $\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_0(t) - \mathbf{d}_i(t)$, $\mathbf{d}_i(t) = \left(\sum_{r=0}^{i} L_r + iD_{\min} \right)[1,0,0]^T$. The following control law is defined for each vehicle:

$$u_i(t) = \mathbf{K} \sum_{j=0, j \neq i}^{N} a_{ij} \mathbf{e}_i(t_k); \ t_k \leq t \leq t_{k+1} \tag{3}$$

where t_k and t_{k+1} are sampling times. By considering time delay $(r(t))$ and actuator lag (Δ), Eq. is represented as:

$$u_i(t - \Delta) = \mathbf{K} \sum_{j=0, j \neq i}^{N} a_{ij} \mathbf{e}_i(t_k - r(t) - \Delta) \tag{4}$$

We assume that maximum m consecutive data may be lost. By defining that $\beta_j(t) = \alpha(t) + (j-1)\eta$, the closed-loop dynamics of convoy will be as follows:

$$\dot{\mathbf{e}}(t) = (\mathbf{I} \otimes \mathbf{A}_1)\mathbf{e}(t) + \sum_{j=1}^{m} \mu_j(t)(\mathbf{H} \otimes \mathbf{A}_2 \mathbf{K})\mathbf{e}(t - \beta_j(t)) \tag{5}$$

The matrix $\mathbf{H}$ is diagonalizable means that there exists a matrix $\mathbf{T}$ such that $\mathbf{T}^{-1}\mathbf{H}\mathbf{T} =$. By defining that $\mathbf{e}(t) = (\mathbf{T} \otimes \mathbf{I})(t)$ and using $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}$, the decoupled closed-loop dynamics of the heterogeneous convoy of vehicles will be as follows:

$$\hat{\mathbf{i}}_r = \mathbf{A}_1 \hat{\mathbf{i}}_r + \lambda_r \sum_{j=1}^{m} \mu_j(t) \mathbf{A}_2 \mathbf{K} \hat{\mathbf{i}}_r(t - \beta_j(t)) \tag{6}$$

where λ_r is eigenvalue of matrix $\mathbf{H}$.

Theorem 1. A convoy of vehicles in presence of random data loss, time delay and actuator lag is internal stable if the following LMIs are satisfied.

$$\dot{\mathbf{E}} = \begin{pmatrix} \dot{\mathbf{E}}_1 & \dot{\mathbf{E}}_{1,2} & \dot{\mathbf{E}}_{2,2} & \dot{\mathbf{E}}_{3,2} & \cdots & \dot{\mathbf{E}}_{m,2} \\ * & \dot{\mathbf{E}}_{1,3} & \dot{\mathbf{E}}_{2,4} & 0 & \cdots & 0 \\ * & * & \dot{\mathbf{E}}_{2,3} & \dot{\mathbf{E}}_{3,4} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ * & * & * & * & \ddots & \dot{\mathbf{E}}_{m,4} \\ * & * & * & * & \cdots & \dot{\mathbf{E}}_{m,3} \end{pmatrix} \prec 0 \tag{7}$$

$$\begin{pmatrix} \mathbf{R}_i & \mathbf{W}_i \\ * & \mathbf{R}_i \end{pmatrix} \succ 0, \quad i = 1, 2, ..., m \tag{8}$$

Proof. By defining the following Lyapunov function and time derivative of it along Eq. .

$$V = \hat{\mathbf{i}}_r^T \mathbf{P} \hat{\mathbf{i}}_r$$

$$+ \sum_{i=1}^{m} \delta_i \left(\int_{t-\eta_i}^{t-\eta_i-1} \hat{\mathbf{i}}_r^T(s) \mathbf{Q}_i \hat{\mathbf{i}}_r(s)ds + \right.$$

$$\left. + \eta \int_{-\eta_i}^{-\eta_i-1} \int_{t+\theta}^{t} \hat{\mathbf{i}}_r^T(s)(\mathbf{R}_i + \mathbf{Z}_i)\hat{\mathbf{i}}_r^T(s)dsd\theta \right) \tag{9}$$

3- String Stability Analysis

The error dynamics of each vehicle is as follows:

$$\tau \dddot{e}_i + \ddot{e}_i = k_1 \left(e_{i-1}(t-\bar{\beta}) - e_i(t-\bar{\beta}) \right)$$

$$+ k_2 \left(\dot{e}_{i-1}(t-\bar{\beta}) - \dot{e}_i(t-\bar{\beta}) \right)$$

$$- k_1 \left(e_i(t-\bar{\beta}) - e_{i+1}(t-\bar{\beta}) \right)$$

$$+ k_2 \left(\dot{e}_{i+1}(t-\bar{\beta}) - \dot{e}_i(t-\bar{\beta}) \right) - k_2 \dot{e}_i(t-\bar{\beta}) \tag{10}$$

Theorem 2. Under the following condition, the string stability of vehicular convoys in presence of random data loss, time delay, and actuator lag is assured.
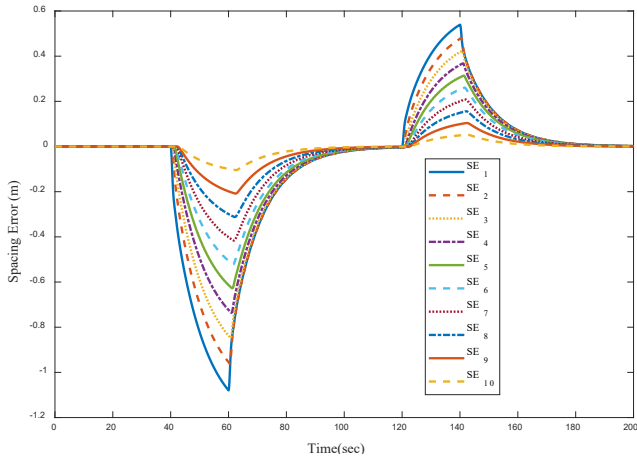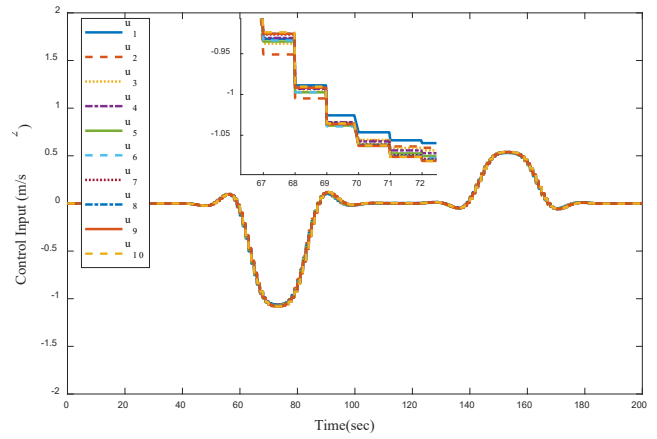
$$k_1 < \frac{5}{4} k_2^2 \tag{11}$$

Fig.2. Spacing error

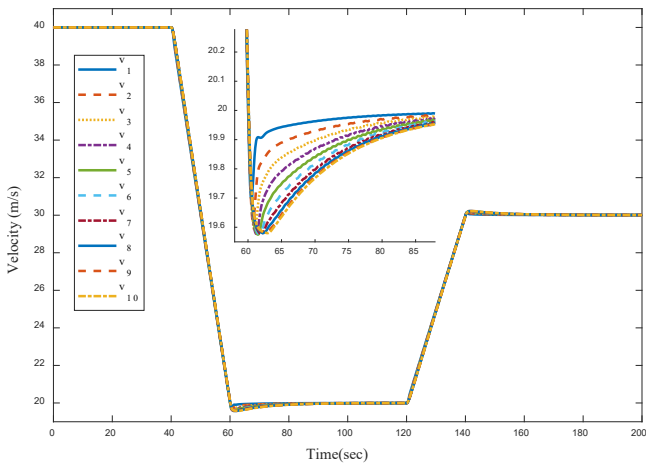**Fig. 4. Upper level control of vehicles**

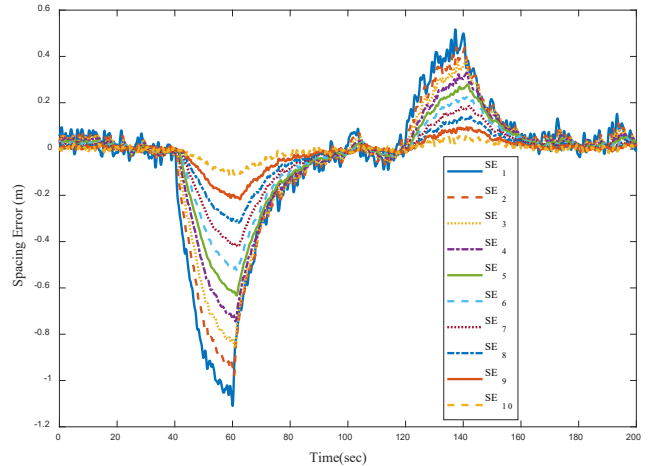**Fig. 3. Velocity of vehicles**

Fig. 5. Spacing error in presence of measurement noise

Proof. By analyzing the string stability condition $|E_i / E_{i-1}| < 1$ in the frequency domain.

Remark. Theorem 2 presents a very interesting result. According to Eq. , the string stability only depends on the control parameters and is independent of data loss, time delay, and actuator lag.

4- Simulation Study

In this section, a convoy of 11 vehicles is considered. Fig. 2 depicts the spacing error of the convoy. According to this figure, the convoy is internal and string stable. Fig. 3 shows that all vehicles track the velocity of the lead vehicle. Fig. 4 shows the upper-level control of vehicles. According to this figure, the maximum acceleration of each vehicle is about $1 \text{m} / \text{s}^2$ therefore, the proposed control law is completely practical and Fig. 5 shows the performance of convoy in presence of measurement noise indicating that the control protocol is robust against noise.

5- Conclusions

In this paper, the internal and string stability of vehicular convoys in presence of random data loss, time delay, and actuator lag was investigated. By employing the Lyapunov-Krassovskii theorem, sufficient conditions on control parameters assuring internal stability were introduced. Afterward, by error propagation analysis in the frequency domain, a condition for string stability was derived. Simulation results verified the merits of the proposed approach.

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