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Fractional calculus approach for bending of viscoelastic plate using two-variable refined plate theory

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ABSTRACT: This paper deals with the time-dependent bending behavior of a rectangular viscoelastic plate based on the two-variable refined plate theory using the fractional calculus approach. The plate is fully simply-supported and is subjected to uniformly-distributed loading and the three-parameter merchant model is used for simulation of viscoelastic behavior. The time-domain governing equations are converted into frequency-domain ones using the Laplace transform and then, these equations are solved by the Navier method. The viscoelastic plate response is obtained using the elastic-viscoelastic correspondence principle so that the response of an elastic equivalent problem is extended into the viscoelastic problem. The results of this study, including plate deflection, and in-plane and transverse strains are compared with the results of the elastic model and the standard merchant model where the comparison of obtained results with the reference ones shows that the proposed approach has good accuracy. Also, the variation of deflection through the plate thickness and the effect of aspect ratio on the results are studied. This study shows that the proposed fractional model has the ability to simulation of both elastic and viscose effects simultaneously which is more compatible with the nature of viscoelastic materials.

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1- Introduction

Fractional calculus is a new field of mathematics in which the integer-order integral and derivative are extended to arbitrary non-integer order ones. This concept has received much attention in various fields of sciences in recent decades. Various definitions are provided for fractional derivative [1]. The static and dynamic analyses of viscoelastic structures have been investigated by many researchers [2-5]. There have also been several kinds of research upon using fractional calculus for the analysis of viscoelastic structures [6-8].

In the present study, the fractional Merchant model is employed for studying the flexural behavior of a viscoelastic plate based on the two-variable refined plate theory [9]. This theory provides accurate results for both thin and thick plates. To solve the obtained time-dependent equations, the Laplace transform method is utilized.

2- Methodology

The Riemann-Liouville fractional derivative definition is employed in this research:

$$D_{RL}^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^{n}}{dt^{n}}\int_{a}^{t}\frac{f(\xi)}{(t-\xi)^{\alpha-n+1}}dt\,, \qquad (n-1>\alpha\geq n, t>a) \tag{1}$$

where α is the order of fractional derivative. The three-parameter fractional Merchant solid model is utilized to define the viscoelastic behavior. The stress-strain relation in this model is defined as:

$$\left(D_{RL}^{\alpha}+\frac{1}{t_1^{\alpha}}\right)\sigma(t)=E_0\left(D_{RL}^{\alpha}+\frac{1}{\tau_1^{\alpha}}\right)\varepsilon(t)$$
$$\tau_1=\frac{\eta}{E_1}\,,\ t_1=\frac{\tau_1}{\sqrt[\alpha]{1+\dfrac{E_0}{E_1}}} \tag{2}$$

where σ and ε are stress and strain and E_0, E_1, τ_1 and η are the viscoelastic material constants. The obtained governing equations are solved using the viscoelastic correspondence principle, the Laplace transform, and the Navier's method. The time-dependent bending and shear components of normal deflection are obtained as follows:

$$w_b(x,y,t)=\frac{16p_0}{D\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{k^2mn}$$
$$\times\left[E_{\alpha}\left(-\frac{t^{\alpha}}{\tau_1^{\alpha}}\right)+\left(1+\frac{E_0}{E_1}\right)\left[1-E_{\alpha}\left(-\frac{t^{\alpha}}{\tau_1^{\alpha}}\right)\right]\right] \tag{3}$$

$$w_s(x,y,t)=\frac{16p_0}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn\left[\frac{D\pi^4}{84}k^2+\frac{5Gh\pi^2}{6}k\right]}$$
$$\times\left[E_{\alpha}\left(-\frac{t^{\alpha}}{\tau_1^{\alpha}}\right)+\left(1+\frac{E_0}{E_1}\right)\left[1-E_{\alpha}\left(-\frac{t^{\alpha}}{\tau_1^{\alpha}}\right)\right]\right] \tag{4}$$

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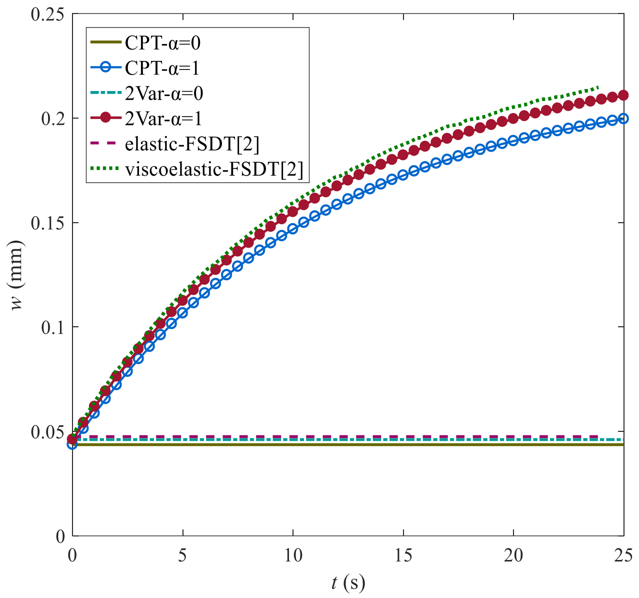


Fig. 1. The deflection obtained by the classical and two-variable plate theories in elastic and viscoelastic cases.

and the in-plane normal strain ε_x and the transverse shear strain γ_{xz} are obtained as:

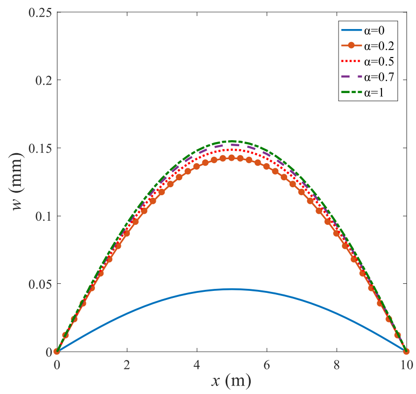
$$\begin{aligned}\varepsilon_x &= 16p_0\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m}{a}\right)^2\frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn}\\ &\times\left[\frac{z}{Dk^2}+\frac{f(z)}{\frac{D\pi^4}{84}k^2+\frac{5Gh\pi^2}{6}k}\right]\\ &\times\left[E_\alpha\left(-\frac{t^\alpha}{\tau_1^\alpha}\right)+\left(1+\frac{E_0}{E_1}\right)\left[1-E_\alpha\left(-\frac{t^\alpha}{\tau_1^\alpha}\right)\right]\right]\end{aligned} \tag{5}$$

$$\begin{aligned}\gamma_{xz} &= \frac{16p_0}{\pi}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}g(z)\left(\frac{n}{b}\right)\frac{\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn\left[\frac{D\pi^4}{84}k^2+\frac{5Gh\pi^2}{6}k\right]}\\ &\times\left[E_\alpha\left(-\frac{t^\alpha}{\tau_1^\alpha}\right)+\left(1+\frac{E_0}{E_1}\right)\left[1-E_\alpha\left(-\frac{t^\alpha}{\tau_1^\alpha}\right)\right]\right]\end{aligned} \tag{6}$$

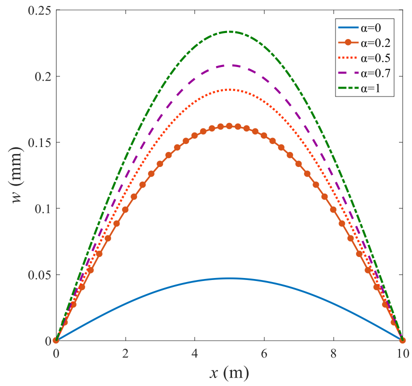
3- Results and Discussion

A fully simply supported square viscoelastic plate with a length of 10m and thickness of 1m under a uniform distributed loading of 10N/m^2 is considered. The obtained deflection of the plate based on classical and refined plate theories is compared with existing results [2] which confirms the accuracy of the proposed approach (Fig. 1).

Fig. 2 illustrates the variation of plate deflection on the x-axis for different values of α in two times of 10s and 50s. Similarly, the variation of transverse shear strain along the plate thickness is depicted in Fig. 3. It is seen that this strain has a parabolic variation through the plate thickness, as expected.

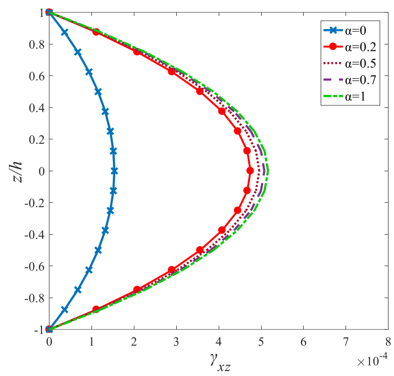


(a)

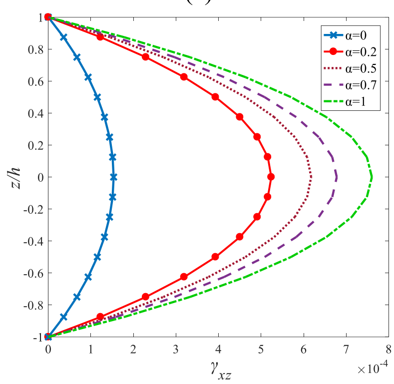


(b)

Fig. 2. The plate deflection considering the two-variable plate theory and different α, a) t=10s and b) t=50s.



(a)



(b)

Fig. 3. The in-plane shear strain considering the two-variable plate theory and different α, a) t=10s and b) t=50s.

4- Conclusions

In this study, the bending analysis of the viscoelastic plate based on a fractional derivative model was conducted. The two-variable plate theory was employed in the formulation which provided accurate results for both thin and thick plates. By using the Riemann-Liouville fractional derivative definition, the formulation was greatly simplified and the viscoelastic behavior was simulated more realistically. Decreasing the fractional derivative order leads to a lower damping property and consequently, the final amount of plate deflection and strain happens sooner. It can be concluded that the approach utilized in this study is very accurate and can be employed to analyze more complex problems, and it also may lead to faster solution procedure due to the fewer number of unknown parameters used in the formulation.

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