



## Variable distance interdigitated electrodes design to improve the performance of cantilever piezoelectric thin films nanogenerators

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**ABSTRACT:** A small-scaled device for ambient energy harvesting is a high voltage cantilever nanogenerator with interdigitated electrodes carrying a tip mass that acts upon the strain induced in the top piezoelectric layer. In this device, more strain gradient over the length, more electric potential in adjacent electrodes depending on the vibration mode shape at which voltage cancellation may occur. In this work, changing the distance between the electrodes proportional to the inverse of strain function, the induced voltage in all the electrodes are equalized that prevents the voltage cancellation. The Euler-Bernoulli beam model is used for the problem and the governing time-dependent equation is derived based on the energy method. Then, the 4th order Runge-Kutta method is used to solve it from which the output voltage is derived for base excitation. The results show that it is possible to increase the voltage by 36% for optimal electrical load by this procedure and for 40% for open circuit conditions. The system coupling is also increased by 10%. Moreover, the results show that the smaller size of electrodes, the higher the output voltage. Whereas, increasing the number of electrodes makes the voltage reduce in contrast with the electric current.

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### 1- Introduction

In the last years, lots of researches have been done on piezoelectric energy harvesters and different electromechanical systems at small scales have been developed [1, 2]. Performance of such devices depends on the structural design, actuation frequency and output voltage for which different mechanisms are suggested. An example of these devices is a cantilevered piezoelectric microbeam covered with a top electrode layer [3]. In these cantilevers, parallel bottom top electrodes for {3-1} mode and the interdigitated electrodes for {3-3} mode of piezoelectric are used. Using the second case regarding the higher value of  $d_{33}$  in comparison with  $d_{31}$  and high output voltage is preferred in many references [4].

Mode shape voltage cancellation is an important issue in piezoelectric nanogenerators. This occurs due to the strain gradient over the length of the piezoelectric layer and makes variable electric potential in interdigitated electrodes and makes the output power decrease. Accordingly, it is noticeable to confine the voltage cancellation and requirements to complicated electrical systems in order to catch the maximum capability of the nanogenerator.

In this work the idea is to design the interdigitated electrodes with variable distance fingers relating to the inverse of strain component along the cantilever and catch the higher voltage from the nanogenerator.

### 2- Vibration Modeling of Cantilever

The strain intensity is variable over the length of the cantilever and its maximum occurs at the root while its minimum occurs at the tip. Proportional to the strain in the piezoelectric layer makes the electric potential to be variable between the adjacent fingers of electrodes. Using variable distance electrodes makes the possibility of producing equal electric potential between the adjacent electrodes and resolve the problem. Accordingly, the voltage cancellation will not occur and electric current will be obtained from the generator without any requirement to complicated electric boards. Moreover, using this idea, it will be possible to cover all the surfaces with interdigitated electrodes and use all the piezoelectric materials in the normal generator. In Fig. 1 the geometrical parameters of interdigitated electrodes for modeling the problem are illustrated. The distance between centers of adjacent electrodes is designated by  $L_E$  and size of the electrodes by  $a$ . Thickness of piezoelectric layer is taken to be  $t_p$ .

In this work, an analytical model is presented for cantilever piezoelectric generators with interdigitated electrodes for which their required relations are derived based on energy principles and the Euler-Bernoulli beam model. Then nanogenerator performance including the managing data is evaluated for the case with a variable distance of the electrodes in comparison with constant distance case. Performance of a piezoelectric in {3-3} mode is given in Eq. (1) for which  $T_1=0$  and  $T_2=0$  regarding Fig. 1 and the electric field will be created only in 3 Direction [5, 6].

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$$\begin{Bmatrix} T_3 \\ D_3 \end{Bmatrix} = \begin{bmatrix} c_{33}^{E*} & -e_{33}^* \\ e_{33}^{S*} & \epsilon_{33}^{S*} \end{bmatrix} \begin{Bmatrix} S_3 \\ E_3 \end{Bmatrix} \quad (1)$$

$$c_{33}^{E*} = \frac{1}{S_{33}^E}, \quad e_{33}^* = \frac{d_{33}}{S_{33}^E}, \quad \epsilon_{33}^{S*} = \epsilon_{33}^T - \frac{d_{33}^2}{S_{33}^E}$$

In this equation  $S_{33}^E$  is the elastic compliance (the inverse of stiffness) in  $x_{33}$  Direction for a constant electric field,  $\epsilon_{33}^T$  is the permeability of the piezoelectric material at a constant stress of  $T$  and  $d_{33}$  is the piezoelectric coefficient. The governing equations for the cantilever piezoelectric nanogenerator are derived using Lagrange equations for the Euler-Bernoulli beam model to derive decoupled differential equations for the deflection of the beam and the electric potential [7-9].

$$M\ddot{r} + C\dot{r} + Kr - \theta v = -B_f \dot{w}_b \quad (2)$$

$$\theta \ddot{r} + C_p \dot{v} + \frac{1}{R_i} v = 0 \quad (3)$$

In these relations  $M$  is the mass  $K$  is the stiffness  $C$  is the damping coefficient  $\theta$  is the coupling  $C_p$  is the capacitance  $v$  is the voltage  $R_i$  is electrical resistance for the electric circuit and  $B_f$  is a function of inertial load against the base excitation. These parameters are considered for the cantilever beam with a tip mass. Two parameters of  $\theta$  and  $C_p$  are given for the whole structure that can be obtained from the following relations:

$$\theta = \sum_{i=1}^N e_{33}^*(i) Q_p \psi_r''(\bar{x}_i) \quad (4)$$

$$C_p = \sum_{i=1}^N \frac{\epsilon_{33}^{S*}(i) A_p}{L_p(i)} \quad (5)$$

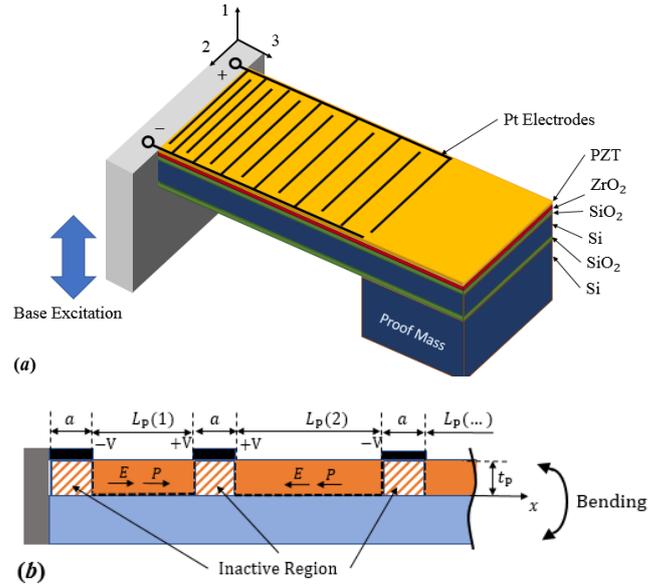
$\psi_r$  is the vibration mode shape function,  $\psi_r'$  and  $\psi_r''$  are the first and second derivatives,  $N$  is the number of elements and  $\bar{x}_i$  is the distance of two electrodes for each element.  $Q_p$  is the moment of area for the piezoelectric layer with respect to the neutral axis of the beam [9]. Regarding Eq. (1), the strain-induced electric field in the piezoelectric element will be equal to:

$$E_3(i) = e_{33}^*(i) Q_p \psi_r''(\bar{x}_i) \quad (6)$$

The electric field and electric potential between two adjacent electrodes are related simply as  $E = -v/L_p$  and therefore the electric potential  $v(i)$  for each element will be proportional to the length of  $L_p(i)$  which is summarized in Eq. (7).

$$L_p(i) = \frac{e_{33}^*(1) \psi_r''(\bar{x}_1) L_p(1)}{e_{33}^*(i) \psi_r''(\bar{x}_i)} \quad i = 1 \dots N \quad (7)$$

In this work, the idea is to select the size of the elements regarding the induced strain gradient in the cantilever during the vibration for constant values of  $e_{33}^*$  and  $Q_p$  to prevent the voltage cancellation. This regards to the connection of all the anodes and cathodes separately together and similar electric potential in all the anode or cathode electrodes the



**Fig. 1. Cantilever nanogenerator with interdigitated electrodes and a proof mass subject to base excitation, (a) variable distance electrodes, (b) side view of cantilever; size of elements, electric field  $E$  and polarization direction  $P$ .**

Eq. (8) will be derived from Eq. (7) to equalize  $v(i)$  for all the elements. Therefore the active length  $L_p$  for each element in interdigitated electrode pattern can be written as Eq. (8) for which the length of the first element is presumed and all the other elements length will be calculated regarding the selected length. This is an assumption for the problem that can be changed to obtain suitable results.

$$L_p(i) = \frac{\psi_r''(\bar{x}_1) L_p(1)}{\psi_r''(\bar{x}_i)} \quad i = 1 \dots N \quad (8)$$

Using this pattern for the electrodes it will be possible to prevent the voltage cancellation and electron transfer in the fingers of anode or cathode electrodes.

### 3- Improvement of Nanogenerators Performance

For parametric analysis of the problem, the effect of electrode distance dependent on the strain intensity is studied and the results are compared with the case of electrodes with constant distances. The geometrical parameters and material properties for the case study are taken from reference [10]. In the section, the structure of the piezoelectric nanogenerator is a multilayer beam with 1 micron thickness of the piezoelectric layer and PZT/ZrO<sub>2</sub>/SiO<sub>2</sub>/Si/SiO<sub>2</sub> sequences of the layers with silicon tip mass and Pt electrodes. Nanogenerator with the configuration of Fig. 1 has an overall length of 7 mm, an active length of 4 mm, and a width of 2 mm.

Results for output power and output voltage are presented for base excitation of 0.25g acceleration at the resonance state and damping coefficient ratio of 0.002. For comparison of the results, the size of the first element is taken to be constant in all the analyses. In Fig. 2 length of the first element  $L_E(1)$

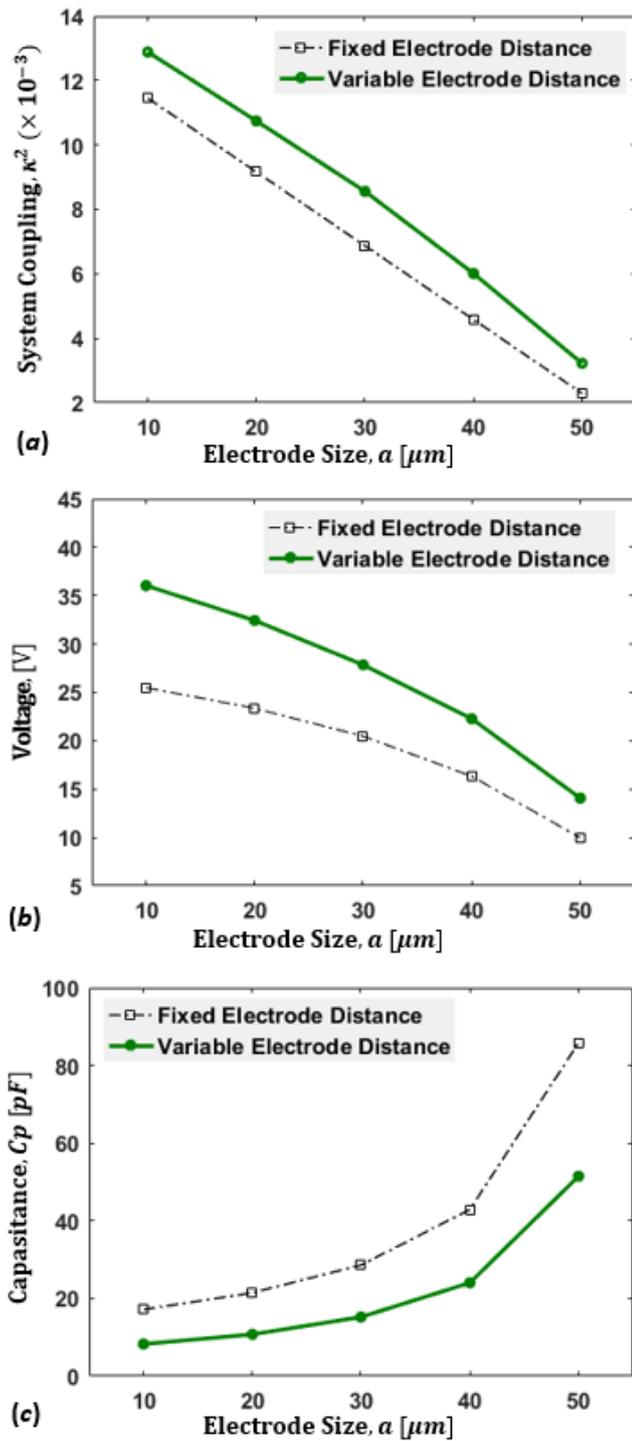


Fig. 2. Nanogenerator's performance versus the electrode size for open circuit condition and  $L_e(1) = 60$  microns, (a) system coupling, (b) Maximum output voltage, (c) nanogenerator's capacitance

is taken to be 60 micrometers and the electrode size of  $a$  is changed from 10 to 50 micrometers. The results for system coupling parameter of  $\kappa^2 = \frac{\theta^2}{KC^2}$ , maximum voltage, and capacitance are given in Fig. 2. For variable distance electrodes, the results in this Figure are given for open-circuit conditions.

For electrode width of 10 micrometers, the magnitude of the output voltage is equal to 36.1 and 25.5 for variable distance and constant distance electrodes respectively. This shows 41% increase in the voltage. Using the present strategy, 40 electrode width of 50 micrometers the voltage is changed from 9.97 to 14.06V that is near to 40% of increasing the voltage. Moreover, the capacitance of the nanogenerator is lower for variable distance electrodes in comparison with conventional designs, and the system coupling is increased by 13% to 41% respectively for electrode width of 10 and 50 micrometers.

#### 4- Conclusions

One of the disadvantages for piezoelectric energy harvesters is the mode shape-dependent voltage cancellation regarding the strain gradient and the corresponding variable electric field that is induced in the structure. In this work, in order to capture the maximum output power from the piezoelectric layer and confine the voltage cancellation in the cantilever nanogenerator, the distance between the electrodes are chosen to be proportional to the inverse of the strain component. Using this configuration, the electric potential for each element related to the nearest electrodes will be constant. The results are presented and compared for variable distance electrodes and constant distance ones. To discuss the relating concepts the results show 41% increase in the efficiency of the system through using the presented strategy. In addition, the results show that for the largest size of electrodes and higher active area of the piezoelectric layer the system coupling gets higher.

#### References

- [1] J. Briscoe, S. Dunn, Piezoelectric nanogenerators—a review of nanostructured piezoelectric energy harvesters, *Nano Energy*, 14 (2015) 15-29.
- [2] S.-G. Kim, S. Priya, I. Kanno, Piezoelectric MEMS for energy harvesting, *MRS bulletin*, 37(11) (2012) 1039-1050.
- [3] Z. Yang, S. Zhou, J. Zu, D. Inman, High-performance piezoelectric energy harvesters and their applications, *Joule*, 2(4) (2018) 642-697.
- [4] C. Wang, Z. Wang, T.-L. Ren, Y. Zhu, Y. Yang, X. Wu, H. Wang, H. Fang, L. Liu, A Micromachined Piezoelectric Ultrasonic Transducer Operating in d33 Mode Using Square Interdigital Electrodes, *IEEE sensors journal*, 7(7) (2007) 967-976.
- [5] A.I.S. 176-, IEEE standard on piezoelectricity, in, IEEE New York, 1987.
- [6] M. Kim, J. Dugundji, B.L. Wardle, Effect of electrode configurations on piezoelectric vibration energy harvesting performance, *Smart Materials and Structures*, 24(4) (2015) 045026.
- [7] N.E. DuToit, B.L. Wardle, Experimental verification of models for microfabricated piezoelectric vibration energy harvesters, *AIAA journal*, 45(5) (2007) 1126-1137.
- [8] N.W. Hagood, W.H. Chung, A. Von Flotow, Modelling of piezoelectric actuator dynamics for active structural control, *Journal of intelligent material systems and structures*, 1(3) (1990) 327-354.
- [9] M. Kim, M. Hoegen, J. Dugundji, B.L. Wardle, Modeling

and experimental verification of proof mass effects on vibration energy harvester performance, *Smart Materials and Structures*, 19(4) (2010) 045023.

[10] N. Elvin, A. Erturk, *Advances in energy harvesting methods*, Springer Science & Business Media, 2013.

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