



## Control of quadrotor by using state-dependent Riccati equation method and analyzing its dynamic performance under wind field

A. Pourmoradi, M. Sabzehparvar\*, A. Ashrafi

Aerospace Engineering Department, Amirkabir University of Technology, Tehran, Iran

**ABSTRACT:** In recent decades, quadrotors are considered, because of the special missions and reducing the cost of flight operation. In this paper, three flight missions are defined to the quadrotor for shooting the special area. Attitude control of quadrotor is analyzed on basis of state-dependent Riccati equation. In the first mission, an experimental sample is taken in order to find the Euler angles for the implementation of routes. The sample quadrotor is on basis of the proportional-derivative controller. For this purpose, results of simulation base on proportional-derivative controller are conducted and the results are validated by state-dependent Riccati equation controller method. In the second and the third missions, the quadrotor is given maneuver by state-dependent Riccati equation method and flies in more complex routes such as square and round to cover more surface. Considering external wind field is the important parameter for the mention missions. The feasibility of these missions related to quadrotor stability and guaranteed security in wind field, for this purpose, the influence of force and moment of the wind field is applied to equations of motion of quadrotor. Dynamic performance of quadrotor is investigated for proportional-derivative, linear quadratic regulator and state-dependent Riccati equation methods encountering wind field.

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### 1- Introduction

UAVs have become more popular in recent decades due to perform impossible missions and reduce operating costs [1]. So far, various linear and nonlinear control methods have been used for better performance. Consideration of external winds, turbulence, disturbances, as well as uncertain dynamic model, has expanded the scope of this research. Sydney et al. Used a nonlinear control method to evaluate the performance of a quadrotor in a turbulent wind field [2]. Li et al. investigated multivariable finite time composite control strategy based on immersion and invariance for quadrotor under mismatched disturbances [3]. Tung Wan et al. investigated quadrotor Performance under gust wind and heavy rain impacts [4]. Since the external wind field is an important factor in quadrotor dynamics, using an applicable control method is essential to perform appropriate missions.

In this study, firstly quadrotor equations of motion are presented, then deferent missions are defined to follow the routes by some control methods, then the external wind equations are added to quadrotor equations in order to evaluate State Dependent Riccati Equation (SDRE), Linear Quadratic Regulator (LQR) algorithms and Proportional-Derivative (PD) controller.

### 2- Quadrotor Dynamic Performance Under Wind Effect

In this section, the wind field effect is applied to the quadrotor in all directions and its dynamic performance is

investigated [5]. To consider the wind effect on quadrotor, the force and moment of wind should be applied to quadrotor's body as an external factor. In this study, the discrete gust model has been used as wind model, then adding the wind to quadrotor equations of motion is explained, eventually, the result is analyzed. According to Fig. 1, to apply the wind effect to the quadcopter, wind speed should first be converted to dynamic pressure and then to force and moment.

The applied force and moment to quadrotor due to wind field are added to main in accordance with Eqs. (1) and (2).

$$\begin{aligned} \ddot{x} &= (\cos \psi \cos \theta) \frac{F_{w_x}}{m} + \\ & (-\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi) \frac{F_{w_y}}{m} + \\ & (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{1}{m} (u_1 + F_{w_z}) \\ \ddot{y} &= (\sin \psi \cos \theta) \frac{F_{w_x}}{m} + \\ & (-\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi) \frac{F_{w_y}}{m} + \\ & (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{1}{m} (u_1 + F_{w_z}) \\ \ddot{z} &= g - \frac{F_{w_x}}{m} \sin \theta + \frac{F_{w_y}}{m} \cos \theta \sin \phi + \\ & \frac{1}{m} (u_1 + F_{w_z}) \cos \phi \cos \theta \end{aligned} \quad (1)$$

\*Corresponding author's email: sabzeh@aut.ac.ir



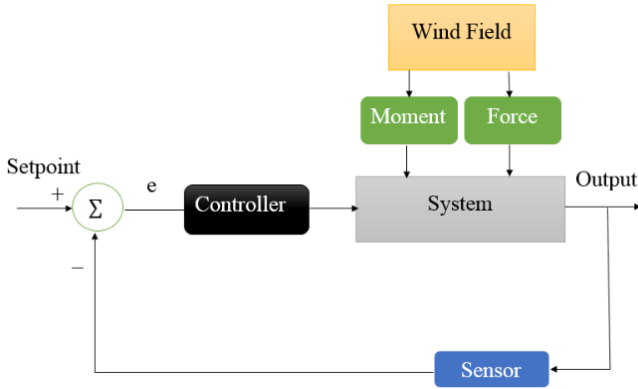


Fig. 1. Diagram of the applied wind to the system

$$\begin{aligned} \ddot{\phi} &= \dot{\theta}\dot{\psi} \frac{I_y - I_z}{I_x} - \frac{J_r}{I_x} \dot{\Omega} + \\ &\frac{(F_{w_{z2}} - F_{w_{z4}})}{I_x} l + \frac{l}{I_x} u_2 \\ \ddot{\theta} &= \dot{\phi}\dot{\psi} \frac{I_z - I_x}{I_y} + \frac{J_r}{I_y} \dot{\Omega} + \\ &\frac{(F_{w_{z1}} - F_{w_{z3}})}{I_y} l + \frac{l}{I_y} u_3 \\ \ddot{\psi} &= \dot{\phi}\dot{\theta} \frac{I_x - I_y}{I_z} + \frac{(F_{w_{x2}} - F_{w_{x4}})}{I_z} l + \\ &\frac{(F_{w_{y1}} - F_{w_{y3}})}{I_z} l + \frac{l}{I_z} u_4 \end{aligned} \quad (2)$$

$F_{w_x}$ ,  $F_{w_y}$ ,  $F_{w_z}$  are the forces due to wind in  $x$ ,  $y$  and  $z$  directions.  $l$  is the quadrotor body's link length from center of gravity to the rotor.

### 3- Quadrotor Attitude and Altitude Control

In this section, firstly attitude control of the quadrotor is investigated. Then position and altitude of the quadrotor are controlled. SDRE algorithm is used for attitude control of the quadrotor, then PD controller is used for  $x$  and  $y$  position control, finally, to control the altitude of the quadrotor, feedback linearization method is selected.

### 4- Simulation of Quadrotor Under Wind Effect

The external wind is applied to the quadrotor with 45 and 30 degrees relative to the  $x$  and  $z$ -axis respectively. SDRE algorithm is used for attitude control of the quadrotor when encountering wind field. Then the results are compared to LQR algorithm and PD controller in similar conditions. Comparison of quadrotor position and altitude for mentioned control methods are illustrated in Figs. 2 to 4. In PD controller and LQR algorithm method, the quadrotor deviates from its desired flight path in  $x$  and  $y$  directions because of the external wind effect, but by the SDRE algorithm method follows the desired route. SDRE and LQR algorithms provide more performance than PD controller in altitude control.

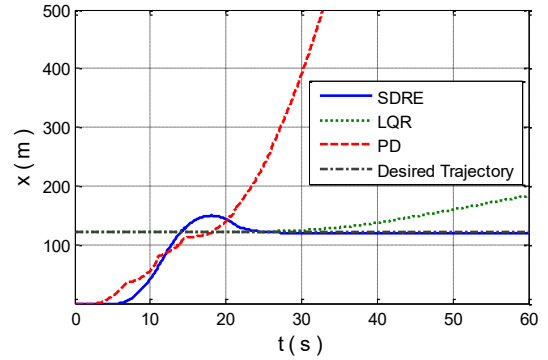


Fig. 2. quadrotor position in  $x$ -direction for three control methods

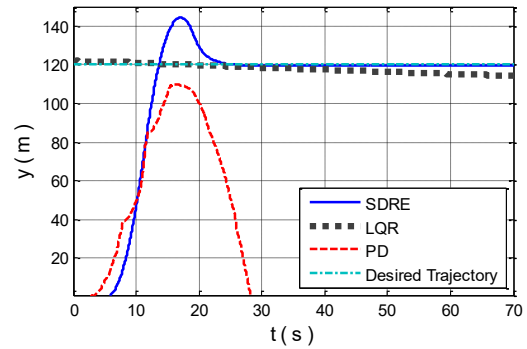


Fig. 3. quadrotor position in  $y$ -direction for three control methods

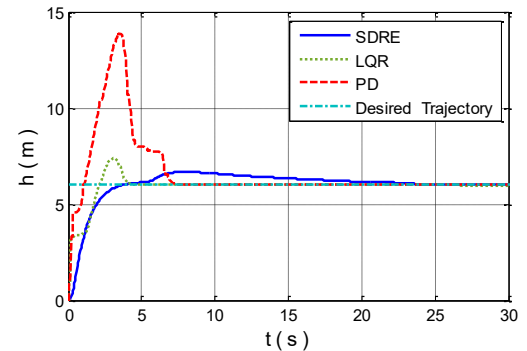


Fig. 4. quadrotor altitude in for three control methods

### 5. Conclusions

In this study, three flight scenarios have been proposed. SDRE algorithm has been implemented to the system. The advantage of this method over linear methods is that the quadrotor is controlled to a certain point without operator's intervention to change Euler angles. It is a closed-loop control system and possible to control when encountering wind field. The quadrotor is stable vertically due to the four rotors but is weak along the horizontal. So, SDRE algorithm can maintain its stability. In an open-loop control system, operators cannot predict the instantaneous forces and moments due to external wind. Therefore, they cannot preserve the quadrotor stability. Another advantage of SDRE algorithm method is complex flight path tracking.

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