# Actuators redundancy resolution scheme with computational time reduction purpose for parallel cable robots with considering the rupture limits of the cables 

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#### Abstract

Cable parallel robots usually require at least one additional actuator force in addition to degrees of freedom to keep the cables in all directions in the workspace, which solves an optimization problem to determine the cable tensile force. In this paper, a convex optimization problem is formulated on a parallel cable robot using optimization conditions through the Karush-Kuhn-Tucker theory and the analytical-iteration method to achieve a minimum force vector of actuators that has less computational time and volume. Where the lower and upper limits of the optimization variables are applied, respectively, to ensure that the cables remain in tension and take into account the saturation limit of the actuators or the rupture limit of the cables (whichever is less), and equal constraints that the relationship between actuator force and force are expressed in the moving platform, defined by the force of the actuators as the sum of the basic solution and the homogeneous solution, -located in the null space of the transpose of Jacobin matrix. Comparison of the results of analytical-iterative solution presented in this paper with numerical algorithms of MATLAB software optimization shows that this method is much faster than these algorithms to converge to the optimal response.


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## 1. Introduction

In parallel cable robots, cable is used instead of rigid arms, which have significant advantages such as lower installation costs, simplicity in design, reduction of inertia and system mass, and so on. However, due to the fact that the cables cannot withstand the compressive force, one or more additional actuators are usually used in the robot to ensure that the cables remain in tension and the system is controlled. Cables are determined by solving an optimization problem. In practice, it is useful to consider redundancy when the computational time of the optimization algorithm is short without the need for very powerful and expensive processors so that these can be used for online applications. Oh and Agrawal [1] have proposed a numerical algorithm for redundancy resolution in which the inputs are all biased to become positive by using the null space contribution of feasible solution and can be directly applied to the robot as a tension force at a minimum-norm solution. Barrette and Gosselin [2] showed that redundancy resolution of cable robots can be considered as an optimization problem with equal and unequal constraints. Zhiwei Cui et al. [3] worked on a new geometric method for optimizing the cable-tension distribution in a cable-driven parallel robots with two degrees of redundancy, in comparison with conventional iteration methods, and also introduced a calculation algorithm for cable-tension polygons based on Graham's scanning. Xinyu
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Geng et al. [4] have proposed a novel measurement index of the magnitude of tensions analytically using the hyper sphere mapping algorithm, which has used the 2-norm quadratic programming of the forces to validate it. Taghirad and Bedoustani [5], instead of numerical methods, have presented an analytical-iterative method based on Karush-Kuhn-Tucker (KKT) theory, in which only the lower limit for the force of actuators is considered and by defining the force of actuators as the sum of the basic solution and the homogeneous solution, located in the null space of the transpose of Jacobin matrix, removes the equal constraint (the relationship between the actuators force and the moving platform force) and makes a significant improvement in the speed of achieving the optimization solution, which is the calculation of the actuators forces. An important point to note is that the use of closed-loop techniques and algorithms has industrial applications when the optimal problem converges to the solution in a short time, i.e. the problem is solved in a short time. The analytical method used in this paper ensures that the solution is achieved in a much shorter time than conventional numerical methods and with much higher accuracy.

In this paper, the nonlinear programming problem using KKT theory is used to solve the optimization problem to generate analytical solutions. Finally, by simulation in MATLAB software, it was observed that the elapsed time in the presented analytical method is much less than the usual numerical optimization methods.

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Fig. 1. 4RPR parallel planar manipulator (a) Schematic diagram of model (b) Kinematic configuration. ([6])

## 2. System Elements and Problem Formulation

This study was performed on a 4RPR parallel plate robot model shown in Fig. 1. In this robot, the moving platform is protected by four limbs with exactly the same kinematic chain.

The instantaneous orientation angle of the points Bi is defined as follows.

$$
\begin{equation*}
\varphi_{i}=\varphi+\theta_{B_{i}} \tag{1}
\end{equation*}
$$

The position of the base points Ai is as follows.

$$
\begin{equation*}
A_{i}=\left[R_{A} \cos \left(\theta_{A_{i}}\right), R_{A} \sin \left(\theta_{A_{i}}\right)\right]^{T} \tag{2}
\end{equation*}
$$

Optimal force distribution in cable robots is very important, which affects the maneuverability and efficiency of the system. The power required to perform the Maneuvers requested is distributed in a direction that uses as little energy as far as possible. Many parallel robots use redundancy in the actuator to solve the problem of singularity and to get it out of control. Due to the fact that cables can only be stretched, the redundancy resolution algorithm makes it possible to calculate the optimal tension force distribution in cables. The optimal problem as follows:

$$
\left\{\begin{align*}
& \min f(\boldsymbol{y})=\|\boldsymbol{\tau}\|_{2}^{2}=\boldsymbol{\tau}^{T} \tau=\left(\boldsymbol{\tau}_{0}+\boldsymbol{B} \boldsymbol{y}\right)^{T}\left(\boldsymbol{\tau}_{0}+\boldsymbol{B} \boldsymbol{y}\right)  \tag{3}\\
& \text { subject to }\left\{\begin{array}{l}
g(\boldsymbol{y})=\boldsymbol{\tau}_{\text {min }}-\left(\boldsymbol{\tau}_{0}+\boldsymbol{B} \boldsymbol{y}\right) \leq 0 \\
h(\boldsymbol{y})=\left(\boldsymbol{\tau}_{0}+\boldsymbol{B} \boldsymbol{y}\right)-\boldsymbol{\tau}_{\max } \leq 0
\end{array}\right.
\end{align*}\right.
$$

$\boldsymbol{\tau}$ denotes the actuator forces vector defined by $\boldsymbol{\tau}=\boldsymbol{\tau}_{0}+B \boldsymbol{y}, \boldsymbol{\tau}_{0}$ denotes the base pseudo-inverse
solution of $\boldsymbol{F}=A \boldsymbol{\tau}$, y can be any arbitrary vector in $\mathbb{R}^{m-n}$ , $\boldsymbol{B}$ orthonormal null space of $A=\boldsymbol{J}^{T}$.

By applying the KKT conditions, the Lagrangian function $L(y)$ is formed by determining the lower bound Lagrangian coefficients $\boldsymbol{\mu}=\left[\begin{array}{llll}\mu_{1} & \mu_{2} & \ldots & \mu_{m}\end{array}\right]^{T}$ and the upper boundary Lagrangian coefficients $\lambda=\left[\lambda_{1} \lambda_{2} \ldots \lambda_{w}\right]^{T}$ as as follows. These coefficients should never be negative.

$$
\begin{align*}
& L(\boldsymbol{y})=f(\boldsymbol{y})+\boldsymbol{\mu}^{T} g(\boldsymbol{y}) \boldsymbol{\tau}_{0}^{T} \boldsymbol{\tau}_{0}+\boldsymbol{\tau}_{0}^{T} \boldsymbol{B} \boldsymbol{y}+\boldsymbol{y}^{T} \boldsymbol{B}^{T} \boldsymbol{\tau}_{0}+  \tag{4}\\
& \boldsymbol{y}^{T} \boldsymbol{B}^{T} \boldsymbol{B} \boldsymbol{y}+\boldsymbol{\mu}^{T}\left(\boldsymbol{\tau}_{\text {min }}-\boldsymbol{\tau}_{0}-\boldsymbol{B} \boldsymbol{y}\right)+\boldsymbol{\lambda}^{T}\left(\boldsymbol{\tau}_{0}+\boldsymbol{B} \boldsymbol{y}-\boldsymbol{\tau}_{\max }\right)
\end{align*}
$$

The KKT necessary conditions for the optimal point yop are in the form of Eqs. (9) and (10).

$$
\begin{gather*}
\frac{\partial L}{\partial \boldsymbol{y}}=2 \boldsymbol{\tau}_{0}^{T} \boldsymbol{B}+2 \boldsymbol{y}_{o p}^{T} \boldsymbol{B}^{T} \boldsymbol{B}-\boldsymbol{\mu}^{T} \boldsymbol{B}+\lambda^{T} \boldsymbol{B}=0 \\
\quad \text { (Gradient Conditions) } \\
\boldsymbol{\mu}^{T} g\left(\boldsymbol{y}_{o p}\right)=0 \quad \& \quad \lambda^{T} h\left(\boldsymbol{y}_{o p}\right)=0  \tag{6}\\
(\text { Switching Conditions })
\end{gather*}
$$

The Hessian matrix of Eq. (4) is the identity matrix that is clearly always positive definite and the quadratic Eq. (4) is always convex. Therefore, a sufficient condition for the global minimum is established, and we reached the answer at each stage. This solution is the global minimum, and there is no need to examine other cases, and the program ends immediately.

The simple expression of Eq. (6) is that for each of the unequal scalar constraints, it must be zero (i.e. the constraint on the boundary of the region is feasible), or the inequality coefficient is zero (i.e. the constraint is located within the boundary of the feasible solution region). With this argument, in each case 8 unknowns of the linear equations will be reduced.


Fig. 2. Block diagram of the closed-loop control topology using an inverse dynamics control.


Fig. 3. Total elapsed time to calculate optimal forces in circular path at analytic-iterative redundancy resolution.

## 3. Results and Discussion

The loop-closed diagram block is shown in Fig. 2, where a decentralized Proportional - Derivative (PD) controller is provided for the closed-loop system.

The average elapsed time in the redundancy resolution method is given in Fig. 3 to follow the desired path in 200 seconds; the average elapsed time in our proposed method was 0.05 ms , which is 63 times shorter than the active-set method.

## 4. Conclusions

In this paper, an analytical method for solving the redundancy resolution problem is described and run on a redundancy parallel cable robot, and the simulation results are extracted. This work is formulated on an optimization problem with equal constraints also unequal constraints on the upper and lower limits, but to simplify problem-solving the equations that express the relationship between the actuator forces and the forces applied to the moving platform have been removed. Nonlinear programming techniques, especially KKT theory, have been used to analyze and achieve analytical solutions. Subsequently, a suitable search algorithm is proposed to check the conditions until the solution is obtained. By extracting the simulation results, it is shown that the average elapsed time in the proposed analytical method in the loop-closed structure, significantly less than common other numerical-iteration optimization methods, and for realtime applications, it can replace numerical methods.

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