



## Fast initial alignment for inertial navigation system based on high order sliding mode observer and Kalman filter

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**ABSTRACT:** The inertial navigation system is a dead reckoning system, thus initial alignment for an inertial navigation system plays an important role in the accuracy of it. In this paper, a novel approach for initial alignment in an inertial navigation system with increased speed and accuracy is proposed. This method has two stages, which integrates the Kalman filter and a high order sliding mode observer. In the inertial navigation system, leveling misalignment angles reach the steady-state faster than the azimuth misalignment angle does, which means the azimuth alignment takes a considerable time for initial alignment. Therefore, in this paper at the first stage estimations of state variables of the system are obtained using the Kalman filter and whenever all variables (except azimuth alignment) reach steady-state, the second stage begins. In the second stage, the estimation which is obtained by the Kalman filter is used as the input to design an equivalent system with unknown inputs for inertial navigation system. A high-order sliding mode observer is then used to estimate the states of a system with an unknown input for estimating the azimuth alignment angle. This method not only increases the speed of estimation but also has comparable accuracy.

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## 1. Introduction

In this paper, a new method is presented based on the Kalman filter and the high order sliding mode observer to estimate the initial alignment. In fact, the convergence error of azimuth misalignment is considered as an unknown input for a system equivalent to the INS error system after applying the Kalman filter and since the leveling misalignment is convergent. It is then used to estimate the new system states of high order sliding mode observer. Bejarano and Fridman [1] present the high order sliding mode observer for simultaneous estimation of system states and unknown input.

In this paper, the method presented by Bejarano and Fridman [1] and its combination with Kalman filter is used to estimate the initial alignment of the inertial navigation system. In summary, the innovations and results of this paper include the following:

- Providing an equivalent model for INS error system, using the initial estimation obtained from the Kalman filter to establish the conditions required for the convergence of the second-class filter, i.e. the high order sliding mode observer.
- Providing a two-step method for estimating high precision initial alignment and fast convergence time.
- Increasing accuracy and reducing estimation convergence time.

- The robustness of the filter provided against system noise and disturbances.

## 2. Methodology

By selecting state variables as such  $x = [\delta v_N, \delta v_E, \psi_N, \psi_E, \psi_D, \nabla_N, \nabla_E, \varepsilon_N, \varepsilon_E, \varepsilon_D]^T$  and considering that only the velocity error in INS error equations can be measured, INS error equations will be in the form of Eqs. (1-3). These equations have been presented in [2].

$$\dot{x} = A_0 x \quad (1)$$

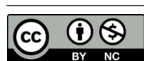
$$y = C_0 x$$

$$A_0 = \begin{bmatrix} F_0 & I_{5 \times 5} \\ 0_{5 \times 5} & 0_{5 \times 5} \end{bmatrix}, \quad F_0 = \begin{bmatrix} 0 & -2\Omega_D & 0 & g & 0 \\ 2\Omega_D & 0 & -g & 0 & 0 \\ 0 & 0 & 0 & -\Omega_D & 0 \\ 0 & 0 & \Omega_D & 0 & -\Omega_N \\ 0 & 0 & 0 & \Omega_N & 0 \end{bmatrix} \quad (2)$$

$$C_0 = [I_2 \quad 0_{2 \times 8}] \quad (3)$$

where  $\delta v$ ,  $r$  and  $\psi$  are the vector of velocity error, position, and orientation respectively,  $\nabla$  is the acceleration

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error vector,  $\varepsilon$  is the gyro drift vector,  $g$  is the acceleration of gravity vector, and  $\Omega$  is the earth rotation rate vector.

The purpose of this section is the presentation of an equivalent model for INS error system described in Eq. (1). The difference between this model and the model presented in Eq. (1) is that in this model the estimation obtained from the Kalman filter is used as the measured input. In fact, in the first step, system state variables of the Eq. (1) are estimated using the Kalman filter and this estimation is used as the output of the new system.

In this model, some system state variables of the Eq. (1) along with the estimation error of the azimuth are considered as unknown inputs for the new system. Thus, the new system model will be as Eq. (4).

$$\begin{aligned} \dot{x} &= Ax + Gd \\ y &= Cx + Hd \end{aligned} \tag{4}$$

where  $x$  is the vector of state variables and  $d$  is the vector of unknown inputs that are considered as Eqs.(5) and (6).

$$x = [\delta v_N \quad \delta v_E \quad \psi_N \quad \psi_E \quad \psi_D \quad \nabla_E]^T \tag{5}$$

$$d = [\psi_D^d \quad \nabla_N \quad \varepsilon_N \quad \varepsilon_E \quad \varepsilon_D]^T \tag{6}$$

In the vector of unknown inputs, the component  $\psi_D^d$  is related to the azimuth convergence error. In fact, as mentioned, the new filter is applied to the equivalent system by applying the Kalman filter after the convergence of leveling misalignment error, the acceleration error and the gyro drift. So, the azimuth estimation error is considered as an unknown input.

According to the Eqs. (1-3), the matrices A and G are as the Eq. (7).

$$A = \begin{bmatrix} 0 & -2\Omega_D & 0 & g & 0 & 0 \\ 2\Omega_D & 0 & -g & 0 & 0 & 1 \\ 0 & 0 & 0 & -\Omega_D & 0 & 0 \\ 0 & 0 & \Omega_D & 0 & -\Omega_N & 0 \\ 0 & 0 & 0 & \Omega_N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The measured output in this system can include all the states of the system because in the first stage, the states of the main system are estimated using the Kalman filter and only the estimation of azimuth error is not available. But this problem is also solved by considering the azimuth estimating error as unknown input. Output matrices are considered as Eq. (8).

$$C = I_6, H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.1 & 1 & 0.3 \\ 0 & 0.6 & 0 & 0.6 & 0.3 \end{bmatrix} \tag{8}$$

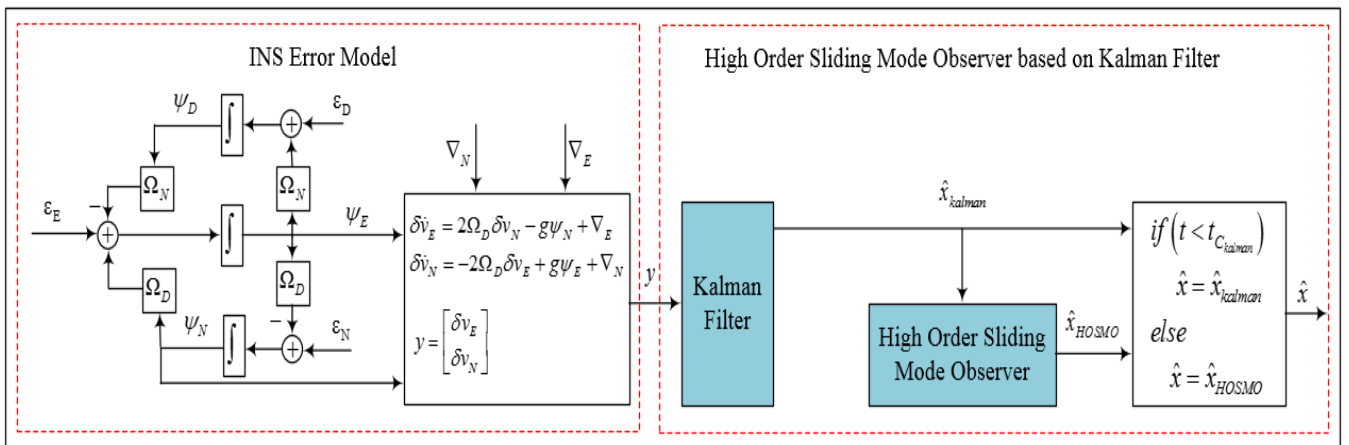


Fig. 1. Overall view of sliding mode observer-based on Kalman filter

### 3. Discussion and Results

In this paper, a new method is presented based on the high order sliding mode observer and Kalman filter for estimating the states of the INS error system. The general view of the observer introduced in this paper is shown in Fig. 1.

To simulate the method presented in this paper, the system expressed in Eq. (1) is expressed as Eq. (9) by considering the noise in the equations and the output of system.

$$\begin{aligned}\dot{x} &= A_0x + w \\ y &= C_0x + v\end{aligned}\quad (9)$$

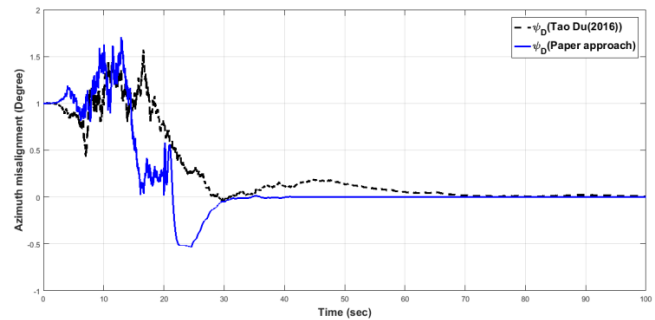
where  $w$  and  $v$  are Gaussian stochastic white noise with mean zero and covariance matrix  $Q$  and  $R$  respectively as Eq. (10). Also in this simulation, local latitude is considered to be 39.9 degrees and initial misalignment error for all three angles is considered to be 1 degree.

$$\begin{aligned}Q &= \text{diag}[(50\mu\text{g}/\text{Hz})^2 \quad (50\mu\text{g}/\text{Hz})^2 \\ &\quad (0.01^\circ/\text{hr}/\text{Hz})^2 \quad (0.01^\circ/\text{hr}/\text{Hz})^2 \quad (0.01^\circ/\text{hr}/\text{Hz})^2 \quad 00000] \quad (10) \\ R &= \text{diag}[(0.1\text{m/s})^2 \quad (0.1\text{m/s})^2]\end{aligned}$$

The results of the filter simulation presented by Du and Yang [3] in the presence of noise and its comparison with the filter presented in this paper, are shown in Fig. 2 to estimate the azimuth error. the convergence time for the filter presented by Du and Yang [3] is about 70 seconds and its accuracy is about 0.02 degrees. While the filter convergence time presented in this paper is about 42 seconds and its accuracy is about 0.003 degrees.

### 4. Conclusions

In this paper, a new method is presented for reducing convergence time and increasing accuracy to estimate the initial alignment of the inertial navigation system. The filter



**Fig. 2. Comparison of azimuth error estimation by the filter introduced in this paper and the filter presented by Du and Yang [3]**

designed in this paper consists of two steps. In the first step, the misalignment error is estimated by the Kalman filter, and after convergence, the second step of the filter, which is a high order sliding mode observer, is used to estimate the azimuth error.

Due to the simulation results, the convergence time of the filter designed in this paper is much less than the filters presented in other studies. One advantage of the filter presented in this paper is its robustness against noise and disturbance.

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#### HOW TO CITE THIS ARTICLE

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