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Reliability analysis of rectangular plate under in-plane tensile loading using continuum damage mechanics theory

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ABSTRACT: In this paper, the reliability of rectangular plates without holes and containing a central circular hole under static tensile load has been studied. To investigate the initiation and evolution of damages, continuum damage mechanics approach together with finite element has been used. Constitutive equations with scalar damage have been obtained for the plate and implemented in finite element code, ABAQUS. To analyze the probability of failure the first/second order reliability methods have been used and then, limit state function and random variables according to the continuum damage mechanics model obtained. The force-displacement curves for various sizes of the hole are obtained. With the addition of a central hole in a plate with a diameter of 2 to 10 mm, failure load is reduced by approximately 60 to 80%, which is consistent with the concepts of stress concentration. Finally, the probability of failure of each plate with different hole sizes is approximated and sensitivity analysis on the coefficient of variation is performed. The reliability of the specimen with a diameter of 10 mm has the lowest value, while the plate without a hole has the highest value and among the random variables, the critical damage is the most effective one in reliability.

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1- Introduction

The Continuum Damage Mechanics (CDM) approach relies on the works of Khachanov [1] and Robotnov [2], which were considered based on uniaxial creep rupture of metals. In recent years there has been much effort to develop damage mechanics in mechanical engineering [3].

In this work, a coupled CDM approach and reliability analysis that can be used to predict plate behavior containing various hole size which is subjected to uniaxial tension loading is formulated within the basic framework of thermodynamics. For this purpose, constitutive equations with scalar damage were implemented in the finite element software, ABAQUS by the subroutine. To calculate the rupture probability, First-Order Reliability Methods (FORM) and Second-Order Reliability Methods (SORM) are used and the limit state functions and random variables will be obtained according to the CDM approach.

2- Continuum Damage Mechanics

In CDM, based on the hypothesis of strain equivalence, the damage variable D by means of the variation in elastic modulus is specified as $D = E_0 - E(D)/E_0$, where E_0 and E are the young modulus of elasticity of the undamaged material and damaged material, respectively. Based on the principle of equivalent strain, Gibbs free energy for isotropic damage can be expressed as follows [4]:

$$\rho\Gamma = \frac{1+\nu}{2E}\frac{\sigma_{ij}\sigma_{ij}}{1-D} - \frac{\nu}{2E}\frac{\sigma_{kk}^2}{1-D} \quad (1)$$

Elastic constitutive equation of the damaged plate under uniaxial loading can be obtained by:

$$\varepsilon_{ij}^e = \rho\frac{\partial\Gamma}{\partial\sigma_{ij}} = \frac{1+\nu}{E}\frac{\sigma_{ij}}{1-D} - \frac{\nu}{E}\frac{\sigma_{kk}}{1-D}\delta_{ij} \quad (2)$$

The energy density release rate in the case of tension $\sigma = \sigma^*$ can be written as:

$$Y = \rho\frac{\partial\Gamma}{\partial D} = \frac{(\sigma^*)^2}{2E(1-D)^2} \quad (3)$$

The constitutive equation of the plastic strain rate is obtained as:

$$\dot{\varepsilon}_{ij}^p = \left(\frac{3}{2}\right)\left(\frac{\sigma_{ij}^D}{1-D}\Big/\left(\frac{\sigma}{1-D}\right)_{eq}\right)\dot{p} \quad (4)$$

where $\dot{p}$ and σ_{ij}^D denotes accumulate plastic strain rate and component of the deviatoric tensor of σ .

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Table 1. Statistical characteristic of material properties of aluminum alloy

Random variables	Mean value	Coefficient of Variation	Distribution type
Critical damage, D_c	0.209	0.08	Normal
Young's modulus, E (GPa)	75	0.05	Normal
Rupture plastic strain, ε_{pR}	0.33	0.05	Normal
Damage threshold plastic strain ε_{pD}	0.031	0.05	Normal

3- Damage Mechanism

According to the thermodynamic framework, the law of damage evolution is defined as [4]:

$$D = D_c \left\langle \left(\varepsilon_p - \varepsilon_{pD}\right) / \left(\varepsilon_{pR} - \varepsilon_{pD}\right) \right\rangle \quad (5)$$

where ε_{pD} and ε_{pR} represent damage threshold plastic strain and plastic strain at rupture, respectively. Then the initiation of a crack or the onset of the fracture was assumed to occur when the damage-associated variable Y attains its critical value Y_c. By denoting the uniaxial fracture stress by σ_R may be expressed also in the form:

$$Y_c = \frac{w_e}{1-D} = \frac{(\sigma_R)^2}{2E(1-D_c)^2} \quad (6)$$

4- Reliability Analysis

The name of the FORM derives from the fact that the limit state function g(x) is approximated by the first-order Taylor expansion which is expressed as follows [5]:

$$g(X) = Z(X) - S(X) \quad (7)$$

where X denotes the random variables, Z and S represent the resistance and load, respectively. So, the probability of failure and subsequent reliability is obtained as follows:

$$\boldsymbol{R} = 1 - P_f = 1 - \Phi(-\beta) = \Phi(\beta) \quad (8)$$

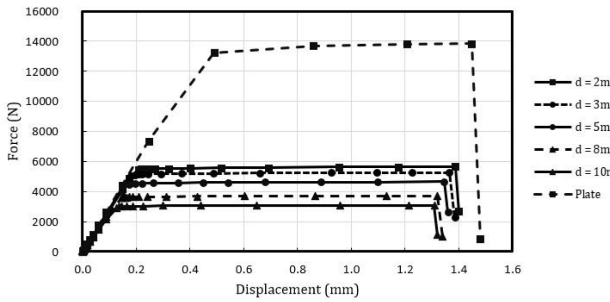
where Φ is the cumulative distribution function of standard normal distribution, P and R is the probability of failure and reliability, respectively. Unlike the FORM, the SORM uses the second order Taylor expansion to approximate the performance function.

5- Methodology

Based on the CDM approach and finite element analysis, the failure function is as follow:

$$g(X) = Y_c - Y \quad (9)$$

Fig. 1. Force-displacement curve of the rectangular plate under in-plane tension loading



According to Section 3, the failure function can be written as:

$$g(X) = \frac{(\sigma_R)^2}{2E(1-D_c)^2} - \frac{(\sigma^*)^2}{2E(1-D)^2} \quad (10)$$

According to Eqs. (5) and (10), in the above limit state function, there are four sources of uncertainty and the vector of random variables is equal to $X = (D_c, E, \varepsilon_{pR}, \varepsilon_{pD})$. For the aluminum 2024 plate under tensile loading, mechanical properties and their statistical information (i.e. the mean value and coefficient of variation), are presented in Table 1.

6- Results and Discussion

Force versus displacement diagrams from typical tensile loading for different hole sizes based on damage models are plotted in Fig. 1. It is clear that with the addition of a central hole in a plate with a diameter of 2 to 10 mm, failure load is reduced by approximately 60 to 80 %, which is consistent with the concepts of stress concentration.

According to damage criteria and failure function, the probability of failure of rectangular plate containing a central circular hole under tension loading is calculated and represented in Table 2.

Finally, sensitivity analysis was performed according and the sensitivity index for each random variable was obtained. As it is obvious in Fig. 2, among the random variables determined in the problem, the critical damage D_c, rupture plastic strain ε_{pR} have the most sensitivity index and other available variables also have the lowest sensitivity index.

Table 2. Comparison of the probability of failure between first-order reliability method and second-order reliability

d (mm)	FORM	SORM
plate	4.6941×10^{-3}	4.9972×10^{-3}
2	4.8951×10^{-2}	5.1994×10^{-2}
3	5.7938×10^{-2}	6.1994×10^{-2}
5	7.9279×10^{-2}	8.3672×10^{-2}
8	8.3898×10^{-2}	8.7268×10^{-2}
10	8.6908×10^{-2}	9.0315×10^{-2}

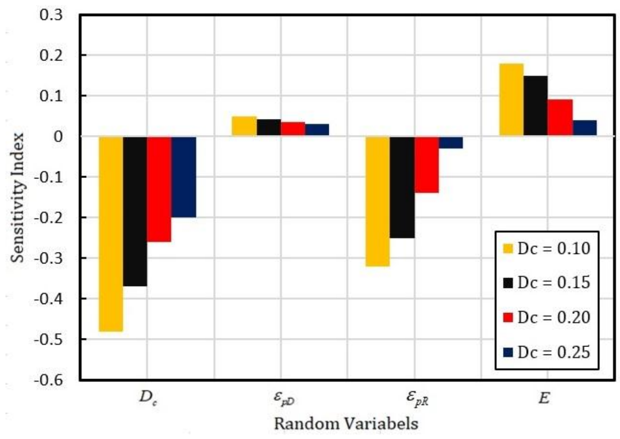


Fig. 2. Sensitivity index of random variables by increasing critical

7- Conclusions

Overall, this study indicates a method for the reliability assessment of a plate containing a central circular hole under in-plane loading using a CDM approach. Reliability analysis was assessed by FORM and SORM. At the first, the constitutive equations of isotropic damage material were applied in the Abaqus software by user subroutine to calculate the stress-strain relationship and the ultimate failure loads of circular hole plate. It was shown that under tension loading, the probability of failure increased by increasing the hole sizes. On the other hand, based on the sensitivity analysis, among the variables, the critical damage and rupture plastic strain have the most sensitivity index.

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